Engineering Applications of Artificial Intelligence 38 (2015) 221–236

Contents lists available at ScienceDirect

Engineering Applications of Artificial Intelligence

journal homepage: www.elsevier.com/locate/engappai

Post Pareto-optimal pruning algorithm for multiple objective optimization using specific extended angle dominance

Sufian Sudeng, Naruemon Wattanapongsakorn *

Department of Computer Engineering, Faculty of Engineering, King Mongkut's University of Technology Thonburi (KMUTT), Bangkok, Thailand

A R T I C L E I N F O

Article history:
Received 20 June 2014
Received in revised form
6 October 2014
Accepted 26 October 2014
Available online 29 November 2014

Keywords:
Multi-objective optimization
Pruning mechanism
Filtering mechanism
Extended dominance
Angle-based pruning algorithm

A B S T R A C T

For the last two decades, significant effort has been devoted to exploring Multi-Objective Evolutionary Algorithms (MOEAs) for solving complex practical optimization problems. MOEAs approximate a representative set of Pareto-optimal solutions and present them to the decision maker (DM). Recently, studies in this area have focused on decision-making techniques in order to help the DM to arrive at a single preferred solution. This paper presents a pruning algorithm which can be applied in the post Pareto-optimal phase to select a subset of robust Pareto-optimal solutions before presenting them to the DM. Our algorithm is called *A*ngle based with *S*pecific bias parameter pruning *A*lgorithm (ASA). Our pruning method begins by calculating the angle between each pair of solutions using an arctangent function. We introduce a bias intensity parameter to calculate a threshold angle in order to identify areas with desirable solutions based on the DM's preference. The bias parameter can be tuned specifically for each objective. We also propose a technique to determine a region of interest using reference point to MOEA/D algorithm which leads to a modified version of MOEA/D (PR-MOEA/D). The experimental results show that our pruning algorithm provides a robust subset of Pareto-optimal solutions for our benchmark problems when evaluating solutions in terms of convergence to optimality.

1. Introduction

Many real-life problems involve several conflicting objectives. Such problems require multi-objective optimization. The conflicting objectives means that a solution that is extreme with respect to one objective requires a compromise in another objective. A classic example of multi-objective optimization problem is the car buying problem. The solution can be expressed in terms of a trade-off between cost and comfort (a two-objective problem). The optimization goal is to find a single best solution out of all of the trade-off solutions, with respect to all objectives.

Typically there is no one best solution to a multi-objective optimization problem, since all solutions involve a compromise between objectives. Instead a single solution, there is a set of trade-off solutions called non-dominated solutions (i.e., no solution dominates or is better than the other solutions in the set). Non-dominated solutions are also called Pareto-optimal solutions. All solutions that are Pareto-optimal constitute the Pareto Set. The objective values of the Pareto set in the objective space constitute the Pareto frontier or Pareto front.

Multi-objective optimization problems can be solved by mathematical modeling approaches that are also effective in finding non-dominated solutions. However, these approaches consume huge computing resources and do not generate multiple solutions in single simulation run. Multi-Objective Evolutionary Algorithms (MOEAs) overcome this limitation. These algorithms imitate natural genetic evolution and apply optimization to a population of solutions. MOEAs can solve multi-objective problems in a reasonable time, and generate multiple solutions that can approximate the entire Pareto-optimal solution in a single run (Barrionuevo, 2011).

Typically, MOEAs have two primary goals: converging to the Pareto-optimal frontier and maintaining a well-spread set of solutions to obtain a good approximation of Pareto-optimal solutions. Most MOEAs try to generate solutions that approximate the entire Pareto-optimal solution front (Adra and Fleming, 2011). However, as indicated by many researchers, this does not help much in choosing a final solution. Consequently, a third goal of converging to the regions that are appealing to the decision maker (DM) has been considered recently. The literature (Branke et al., 2001; Bechikh et al., 2011; Sindhya et al., 2011; Sinha et al., 2013; Soylu and Ulusoy, 2011) stresses the importance of and examines the methods for converging to the interesting portions of the Pareto front, where “interesting” is defined in terms of the DM's preferences.

* Corresponding author. Tel.: +662 470 9089; fax: +662 872 5050.
E-mail address: naruemon@cpe.kmutt.ac.th (N. Wattanapongsakorn).

http://dx.doi.org/10.1016/j.engappai.2014.10.020
0952-1976/

In real-world applications, the DM is not interested in the whole Pareto front since often the final decision is just a single solution. The main goal of MOEAs is to assist the DM in selecting the final alternative which satisfies most or all of his/her preferences. To simplify the decision making task, the DM can incorporate his/her preferences into the search process. These preferences are used to guide the search towards the preferred parts of the Pareto front from the DM's perspective (Xiong et al., 2012). These parts are called the region of interest (ROI). Several strategies have been developed to model the DM's preference information including weight preference (Zitzler et al., 2007; Friedrich et al., 2013), weight reference point (Basseur et al., 2012), solution ranking (Deb et al., 2010) and so on. Some strategies have been developed in order to perform this in the post Pareto-optimal phase such as those presented in Kulturel-Konak et al. (2008) and Wattanapongsakorn and Leesutthipornchai (2013). However, many preference strategies still have difficulties in choosing the final best solution. In addition, most of the preference strategies require some level of background knowledge from the DM. Considering these issues, in this paper we propose a new pruning mechanism that can filter out undesired solutions and provide solutions to the DM that match his or her expressed preferences.

The contributions of our research can be summarized as follows:

1. We propose a new pruning algorithm which can be used in post-Pareto optimal phase.
2. The algorithm has specific bias parameters that allow the DM to prioritize each objective according to his/her preferences.
3. We offer the possibility of pruning more solutions inside and outside the region of interest, which is identified by the DM.

2. Multiple objective optimization

Real-world problems commonly require the simultaneous consideration of multiple performance measures. Most often, the multiple objectives are in conflict and compete with each other. The DM has to choose an individual solution based on certain preferences or priorities for different objectives. In its general form, a multi-objective optimization problem can be formulated as follows:

"Minimize" $z=f(x)$

Subject to $x \in X$

where,

- $f(x)=(f_1(x), \ldots, f_i(x), \ldots, f_p(x))^T$ is p-vector of objective functions.
- $x=(x_1, \ldots, x_n)^T$ is decision vector.
- $X \subseteq R^n$ is feasible decision space.
- $z=f(x)$ is objective vector and $Z=f(X)$ is solution space.

$*^T$=Transpose

A solution x that satisfies all constraints and variable bounds is a *feasible solution*. Otherwise it is called an *infeasible solution*. *Feasible space* is a set of all feasible solutions. The objective function $f(x)=(f_1(x), f_2(x), \ldots, f_M(x))^T$ defines a multi-dimensional *objective space*.

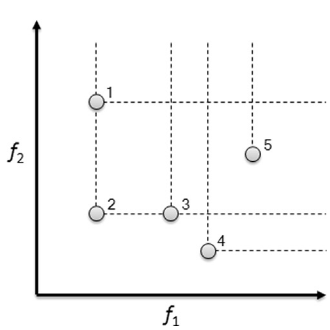

Fig. 1. Concept of dominance.

2.1. Pareto-optimality

In multi-objective problems, the concept of "dominance" is used to determine if one solution is better than others. A solution x is said to dominate a solution y if the following two conditions are true: (1) x is no worse than y in all objectives and (2) x is better than y in at least one objective. In this case y is said to be "dominated" by x, or alternatively, x is said to be "non-dominated" by y. Fig. 1 illustrates the concept of dominance in a two-objective minimization problem. Since both functions are to be minimized, the following dominance relationships can be observed: solution 2 dominates solutions1, 3 and 5; solution 3 dominates only solution 5 and solution 4 dominates only solution 5. Furthermore, solutions 2 and 4 are non-dominated because there is no solution that dominates them.

Note that even if solution 2 is equal in one objective to solutions 1 and 3, it still dominates them, given the definition of dominance. The non-dominance relationship determines the concept of Pareto optimality. A solution is said to be Pareto optimal if it is not dominated by any other solution. In other words, a Pareto-optimal solution cannot be improved in one objective without losing quality in another one. In the example, solutions 2 and 4 are *Pareto-optimal solutions*. The set of all solutions that are Pareto-optimal constitute the *Pareto set*. The objective values of the Pareto set in the objective space constitute the *Pareto front*.

2.2. Multi-objective Optimization with Evolutionary Algorithms (MOEAs)

The term evolutionary algorithm (EA) stands for a class of stochastic optimization methods that simulate the process of natural evolution. The origins of EAs can be traced back to late 1950s (Goldberg, 1989). Since then, many evolutionary algorithms have been developed. The EA approaches operate on a set of candidate solutions. This set is modified by the two principles of selection and variation. Selection imitates the competition for reproduction and resources among living beings. The other principle, variation, imitates the natural capability of creating new living being by means of recombination and mutation. Early attempts to use EA in multi-objective optimization problems focused on classical approaches (e.g., weighted-sum or ε-constrained). Soon, however, researchers started to develop novel algorithms that exploited the powerful concepts behind EA. The Vector Evaluated Genetic Algorithm (VEGA) developed in 1984 is considered the first MOEA. After VEGA, the next decisive milestone was proposed by Goldberg (Goldberg, 1989) with the use of Pareto optimality as the fitness criterion. In this approach, the population is ranked in terms of fronts (Pareto ranking). The non-dominated solutions obtain the highest rank (associated with highest fitness). The next front is given the second highest rank and so on.

Recently, many MOEAs have been introduced such as:

- MOCell (Nebro et al., 2009): a cellular based MOEA
- AbYSS (Nebro et al., 2008): multi-objective scatter search
- MOPSO (Durillo et al., 2009): a particle swarm based MOEA
- Hybrid MOEA (Sindhya et al, 2013): multi-algorithm based MOEA
- MOEA/D-M2M (Liu et al., 2014): an aggregation based MOEA

In addition, in Wang et al. (2013), the authors categorized MOEAs based on their fitness schemes including Pareto dominance based MOEAs, aggregation (or weight) based MOEAs, epsilon dominance based MOEAs, and indicator based MOEAs.