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# Proportional derivative fuzzy control supplied with second order sliding mode differentiation

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ABSTRACT

The fuzzy logic controller (FLC) has the ability of handling parametric uncertainties and external perturbations for unknown systems. A regular structure for a FLC is the proportional derivative (PD) form. The proportional derivative fuzzy controller (PDF) could be seen as a variable gain PD controller. Despite this characteristic, the most common drawback for any PD controller, with unknown dynamics or even with unmodeled dynamics is the error signal differentiation. In this manuscript this disadvantage was overtaken implementing the super-twisting algorithm (STA) as a robust exact differentiator (RED). The information provided by the STA was injected into the PDF to enhace its performance. In this study, the stability of the nonlinear system under the fuzzy super twisting PD controller (FSTPD) in closed loop was analyzed using the concept of the second Lyapunov's method. Numerical simulations were designed to show the effectiveness and advantages of the proposed FSTPD over the classical PD structure supplied with the STA and a PDF with the derivative part obtained by a linear filter. A first example to stabilize a simple pendulum was developed applying the FSTPD. A second example for solving a tracking control problem was designed for a robot manipulator with six degrees of freedom. In both cases, the FSTPD showed better performance and a significant reduction of the control energy.



## 1. Introduction

Proportional Derivative (PD) controllers have become the most successful approach implemented in industrial applications. A PD provides a simple structure and a certain degree of accuracy for systems described by a suitable mathematical model. PD controllers can reduce the overshoot, rise time, settling time and it can force better steady state behavior of the system in closed loop (Khalil, 2002; Kuo, 2005).

Since the first successful application of fuzzy sets in control systems (Mohan and Patel, 2002; Zadeh, 1965), fuzzy logic control (FLC) in engineering has attracted the attention of many researches. The main idea working with FLC is devoted to control complicated nonlinear systems with a fixed set of control rules, usually derived from the knowledge of an expert. Several works regarding the application of FLC as a PD structure have been presented obtaining remarkable results (Su et al., 2004; Mudi and Pal, 1999), (Malki et al., 1994). The stability analysis of FLC have been made in terms of bounded input–bounded output (BIBO) stability and Lyapunov theory (Mohan and Patel, 2002; Tang et al., 2001).

A classical PD controller is composed by proportional and derivative terms. Most of the applications where a PD controller is involved assume the complete access to the state vector and the full description of the mathematical model. However, these assumptions are difficult to fulfill in many real situations. Among others, working with mechanical systems, where the available output is usually described by the position of the nonlinear system, a tachometer must be implemented to obtain the velocity of the system and then be able to apply a PD controller (Su et al., 2004). Unfortunately, in other real applications, the on-line measurement of the signal error derivative is not available or the cost of implementation is raised. That means, inherit to PD controllers, there exists an associated problem, the signal differentiation.

Several attempts have been proposed to deal with the problem of signal differentiation (Su et al., 2004; Dridi et al., 2010; Mboup et al., 2007; Levant, 1998). The most useful tools are based on linear filters and observers (Yu and Ortiz, 2005). However, the main problem arises when the signal to be differentiated contains high frequency disturbances or the mathematical description of the signal is unknown (Levant, 1998). Luenberger state estimators are also applied but the exact model of the signal is a requirement for implementing this approach (Dridi et al., 2010). Moreover, the observer parameters are not tuned to reduce sensitivity to measurement noises or perturbations (Atassi and Khalil, 2000). Another approach was given in terms of an $H^{\infty}$ problem for LTI systems, but it offers only sufficient

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conditions with a solution of a LMI that defines a minimization problem (Dridi et al., 2010). For the case of nonlinear systems with random noises or perturbations, the Kalman filters are the principal solution to obtain the tracking error derivative (Tsai et al., 2007).

When the mathematical description of a plant is unknown or presents parametric uncertainties, a PD controller could become inaccurate if the method selected to obtain the derivative is not properly chosen. The problem of signal differentiation when the signal contains nonlinearities and bounded noises has been solved by means of nonlinear differentiations. In Su et al. (2004), a complete PD controller with a nonlinear differentiator is applied to control a permanent-magnet synchronous motor but the mathematical model is required. For systems with parametric uncertainties, unknown dynamics and noise in the available output, sliding mode control (SMC) is a solution for the problem of signal differentiation (Levant, 1998). The SM offers attractive characteristics such as finite time convergence and robustness again parametric uncertainties (Utkin, 1992). The so-called second order Sliding Modes theory (SOSM) is a powerful tool for the problem of control and state estimation (Levant, 2007). In particular, the super-twisting algorithm (STA) has been applied to deal with nonlinear systems with parametric uncertainties and bounded perturbations. The STA has been studied in several articles and it has been applied as a controller (Dávila et al., 2009), state estimator (Davila et al., 2005) or as a RED (Levant, 1998). Applied as a RED, the STA is capable to reproduce the derivative of an unknown nonlinear trajectory no matter the presence of high frequency and bounded perturbations in finite time.

The stability of SOSM had been analyzed in terms of geometric characteristics (Davila et al., 2005). However, the advances in non-smooth Lyapunov theory have allowed the development on new non-smooth functions to proof the convergence of SOSM under the Lyapunov approach (Clarke et al., 1998; Polyakov and Poznyak, 2009). The work presented in Moreno and Osorio (2012) established a simple quadratic Lyapunov function to obtain sufficient conditions for adjusting the gains of the STA in order to obtain finite-time convergence in presence of bounded perturbations.

Several studies have included nonlinear techniques such as FLC and neural networks to obtain a nonparametric representation of the uncertain nonlinear system (Mudi and Pal, 1999; Poznyak et al., 2006; Sung-Kwun et al., 2009). In this paper, the stability analysis of a fuzzy PD controller in closed loop with the STA as a RED is studied. Sufficient conditions for guarantying the stability of the closed loop system are obtained with a non-smooth Lyapunov function. Under certain kind of high frequency bounded perturbations, the proportional derivative fuzzy controller supplied with the STA (FSTPD) provided practical stability characterized by a boundary layer around the origin. Numerical simulations are given to show the advantages of the proposal presented in this work in comparison with other kind of signal controllers and linear differentiation algorithms.

The rest of the paper is organized as follows, in Section 2 the class of nonlinear systems to deal with it is introduced, then the STA as a differentiator is described. In this section, the extended system that incorporates the STA to estimate the derivative of the signal error is given. In Section 3 the main result is summarized in a theorem. Numerical results are presented in Section 4. Finally in Section 5, the conclusions are given.

## 2. Super-twisting fuzzy PD controller

### 2.1. Class of nonlinear systems

Consider the nonlinear system described by the following second order nonlinear differential equation:

$$
\begin{aligned}
&\ddot{z}(t)=f(z(t),\dot{z}(t))+g(z(t))u(t)+\eta(\dot{z}(t),z(t),u(t),t)\\
&y(t)=z(t)\\
&z(0)=z_0 \text{ and } \dot{z}(0)=z_{d0} \text{ given}\\
&z_0,\ z_{d0}\in\Re^n
\end{aligned}
\tag{1}
$$

Here $z\in\Re^n$ and $\dot{z}\in\Re^n$, $z(0)$ and $\dot{z}(0)$ are the initial conditions for the differential equation. The drift term $f:\Re^{2n}\to\Re^n$ is a Lipschitz function and the input associated term $g:\Re^n\to\Re^{n\times n}$ is bounded as it will be described later. The nonlinear function $\eta:\Re^{2n+1}\to\Re^{3n}$ represents some uncertainties affecting the nonlinear system satisfying

$$
\|\eta\|^2\le\eta_0+\eta_1\left\|\begin{matrix}z\\ \dot{z}\end{matrix}\right\|^2,\quad \eta_0,\eta_1\in\Re^+
\tag{2}
$$

The signal $y\in\Re^n$ is the available output vector. The control action is represented by $u\in\Re^n$. The class of systems considered in (1) is a rough generalization of many mechanic, electromechanical, electric, thermodynamic and hydrodynamic systems. The system presented in (1) can be represented as (with the selection of $x_a=z$ and $x_b=\dot{z}$)

$$
\begin{aligned}
&\dot{x}_a(t)=x_b(t)\\
&\dot{x}_b(t)=f(x(t))+g(x_a(t))u(t)+\eta(x(t),u(t),t)\\
&y(t)=Cx(t)
\end{aligned}
\tag{3}
$$

where $x=[x_a^\top\ x_b^\top]^\top\in\Re^{2n}$ and $C=[I_{n\times n},0_{n\times n}]$.

Throughout the paper, the following assumptions are assumed to be fulfilled.

**Proposition 1.** *The nonlinear function $f(\cdot)$ is unknown but satisfies the Lipschitz condition*

$$
\|f(x)-f(x')\|\le L_1\|x-x'\|,\ \ \forall x,\ x'\in\Re^{2n},\ L_1\in\Re^+
\tag{4}
$$

**Proposition 2.** *The nonlinear system (3) is controllable, therefore the function $g(x)$ is known and it satisfies*

$$
0<g^-\le\|g(x_a)\|\le g^+<\infty,\quad \forall x_a\in\Re^n,\ g^-,g^+\in\Re^+
\tag{5}
$$

*By this assumption, the matrix $g(x_a)$ is invertible $\forall t\ge0$.*

**Proposition 3.** *The control input belongs to the set $U^{adm}$ defined as*

$$
U^{adm}:=\{u:\ \|u\|^2\le u_0+u_1\|x\|^2\}
\tag{6}
$$

*with $u_0,\ u_1\in\Re^+$. The previous condition includes several control techniques such as classical PD controllers and even discontinuous controllers such as sliding modes.*

### 2.2. Nonlinear reference system

The problem considered in this paper was to complete the trajectory tracking between the states of (1) and the stable reference model given by

$$
\begin{aligned}
&\ddot{z}^*(t)=h(\dot{z}^*(t),z^*(t)),\quad z^*(0),\quad \dot{z}^*(0)\text{ are given}\\
&y^*(t)=z^*(t)
\end{aligned}
\tag{7}
$$

where $h(z^*,\dot{z}^*)$ $(z^*\in\Re^n)$ is a Lipschitz function. Again, system (7) can be transformed using the state space method with the change of variables $x_a^*=z^*$ and $x_b^*=\dot{z}^*$. The reference system (7) has a stable equilibrium point and by the converse Lyapunov theorem (Khalil, 2002), one can ensure that the system in the new coordinates $x^*=[(x_a^*)^\top\ (x_b^*)^\top]^\top$ satisfies

$$
\begin{aligned}
&\|x^*(t)\|^2\le X_+^*,\quad X_+^*\in\Re^+,\quad \forall t\ge0\\
&x^*=[(z^*)^\top,(\dot{z}^*)^\top]^\top
\end{aligned}
\tag{8}
$$

then, the next inequalities are assumed to be valid

$$
\|f(x^*)-h(x^*)\|_\Lambda^2\le h^+,\quad \Lambda=\Lambda^\top>0,\quad \Lambda\in\Re^{n\times n}
\tag{9}
$$

$\forall x^*\in\Re^{2n}$ solution of (7), the last inequality is valid because functions $f(x^*)$ and $h(x^*)$ are Lipschitz (continuous) functions and (7) has a stable equilibrium point.