Engineering Applications of Artificial Intelligence 30 (2014) 189–198

Contents lists available at ScienceDirect

Engineering Applications of Artificial Intelligence

journal homepage: www.elsevier.com/locate/engappai

Stability and chaos analysis of a novel swarm dynamics with applications to multi-agent systems

Swagatam Das [a,*], Debdipta Goswami [b], Sarthak Chatterjee [b], Sudipto Mukherjee [b]

[a] *Electronics and Communication Sciences Unit, Indian Statistical Institute, Kolkata, India*
[b] *Department of Electronics and Telecommunication Engineering, Jadavpur University, Kolkata, West Bengal, India*

ARTICLE INFO

Article history:
Received 24 May 2013
Received in revised form
30 September 2013
Accepted 20 December 2013
Available online 5 February 2014

Keywords:
Swarming
Attractant–repellent profile
Lyapunov exponent
Chaos
Stability
Limit cycle

ABSTRACT

This paper presents a novel swarm dynamics and illustrates its applications in automated multi-agent systems. The motion of the particles of the swarm in a particular landscape is governed by an attractant–repellent profile, which has an intimate linkage with the distance separating the particles. Following standard stability and chaos analysis procedures, it is demonstrated that the dynamics indeed simulates a swarm. We adopt a Lyapunov-function based stability and chaos analysis procedure to this effect. The parameterized conditions for which the dynamics exhibits chaotic characteristics are also investigated. Finally, the swarming dynamics is applied to a practical problem, thus elucidating how the proposition can be of use in a real-life situation. Since the dynamics rests on the values of certain parameters, we can control the areas in which we want to use the dynamics by controlling these parameters. The proposed dynamics will be shown to produce convergent, limit cyclic and chaotic behavior. This swarming dynamics can therefore be put to myriad uses depending on the application that is required.

1. Introduction

Swarm behavior, or swarming, can be loosely defined as the collective intelligence exhibited by living or non-living entities, the prime characteristic of which is the *en masse* migration of the individuals in question towards a particular direction. Numerous biological systems exhibit swarming behavior, the most pivotal of them being that exhibited by birds, insects and fish. When applied to inanimate entities, the term may be applied to the control of automated systems of multiple robots (Buhl et al., 2006; Egerstedt and Hu, 2001), programmed vehicles on sea, air or land (Fax and Murray, 2004; Gil et al., 2008a), mechanized tracking (Gil et al., 2008b), rendezvous (Dimarogonas and Kyriakopoulos, 2007), coverage supervision on mobile sensing networks (Cortes et al., 2004), and so on.

The inherently biological phenomenon of swarming (Breder, 1954; Warburton and Lazarus, 1991; Okubo, 1986; Grünbaum and Okubo, 1994; Mogilner and Edelstein-Keshet, 1999; Durrett and Levin, 1994; Gueron and Levin, 1995) has greatly intrigued physicists (Levine and Rappel, 2001; Vicsek et al., 1995; Czirok et al., 1996, 1997; Czirok and Vicsek, 2000; Shimoyama et al., 1996) and mathematicians alike. Computer scientists have also latched onto these patterns by working out efficient optimization algorithms that mimic the real-time behavior of biological swarms. Out of these efforts, algorithms like the ant colony optimization (ACO) algorithm (Bonabeau et al., 1999), the particle swarm optimization (PSO) algorithm (Kennedy et al., 2001; Clerc and Kennedy, 2002) and the bacterial foraging optimization algorithm (BFOA) (Passino, 2002) have evolved.

Since the present work bases itself on the behavior of swarms, a brief review of the various sources and characteristics of swarming behavior present in the currently published literature would not be inappropriate. Extensive amounts of research have been carried out in the field of swarm dynamics, and this has opened up several new and interesting avenues in the aforementioned discipline.

Gazi and Passino (2004) considered an M-individual swarm in an n-dimensional Euclidean space and then modeled the behavior of the constituent particles of the swarm based on the nature of an attractant–repellent profile, or the "$\sigma-$profile", as it was referred to by them. Liu and Passino (2004) improved the social foraging swarm profile by modifying it to suit the presence of sensor errors, and even the presence of a considerable amount of various types of noise on the profile, all the while maintaining the robust and cohesive nature for which the profile is known. Leonard and Fiorelli (2001) added support to the above claims by showing allied results based on artificial potentials and virtual leaders for agents with point-mass dynamics. Li (2008) carried out a rigorous study on the stability characteristics of a swarm with general

* Corresponding author.
E-mail addresses: swagatam.das@isical.ac.in (S. Das), eigenvalue_debdipta@yahoo.in (D. Goswami), sarthak.chatterjee92@gmail.com (S. Chatterjee), sudipto.dip15@gmail.com (S. Mukherjee).

0952-1976/$ - see front matter © 2014 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.engappai.2013.12.014

directed and weighted topology. Liu et al. (2003) derived conditions for one-dimensional asynchronous swarms to achieve collision-free convergence even in the presence of sensing delays and asynchronism. For the members of a swarm to behave in a manner that justifies the use of the swarming dynamics to solve real-life practical problems, collision avoidance is a critical issue. For example, it is practically useless to propose a dynamics, the constituent particles of which agglomerate after a few iterations. Agglomeration is a very potent issue to be considered, especially when the simplifying assumption of the particles being point masses is not rigorously maintained. Finite particle size does tend to cause problems, the solutions of which are clearly addressed by Liu et al.

Chaos is formally defined as aperiodic long-term behavior in a deterministic system that exhibits sensitive dependence on initial conditions (Strogatz, 1994). The term deterministic means that the system has no random or noisy inputs or parameters. The irregular behavior arises from the system's nonlinearity, rather than from the noisy driving forces. The systematic mathematical study of chaotic systems has hugely developed during the second part of the 20th century (Chen and Dong, 1998; Andrievskii and Fradkov, 2004). Frank et al. (1990) conducted an extensive study of neural network systems, the complex interconnections which help human beings to think. A time-series analysis of chaotic behavior has revealed an intimate linkage of chaos with epileptic seizures. Chen (1988) showed proof of evidence of the presence of low-dimensional strange attractors in several empirical monetary ensembles. Monetary growth is described by a continuous-time deterministic model with delayed feedback. Phase transitions from periodic to chaotic motion are shown.

Chaos analysis in artificial neural networks (ANNs) has occupied significant amounts of space in academia in recent years. Along the lines of Das et al. (2012), we propose a brief survey of the latter. Chen and Aihara (1995) proposed a transiently chaotic neural network (TCNN) as an approximate method for solving combinatorial optimization problems. A transiently chaotic dynamics has been introduced into the neural network. Unlike conventional neural networks with point attractors only, the proposed neural network has richer and more flexible dynamics, so that it can be expected to have higher ability of searching for globally optimal or near-optimal solutions. Nozawa (1992) introduced a neural network model as a globally coupled map. Nozawa's model takes a cue from the network model of Hopfield, possessing a negative self-feedback connection. This model analyzes information in terms of the multitude of maps acting on the constituent nodes of the network, and gives a novel way by which information may be processed. The model is tested on information search using abstruse keywords and the classic traveling salesman problem (TSP). A chaotic approach to solving optimization problems using the technique of simulated annealing has been given by Chen and Aihara (1995). This is different from the model proposed earlier by Aihara et al. in that a negative self-coupling added to the former and thereafter, slow removal of the same produces a transient chaos. This transient chaos is used for combing or self-organizing the search space, thereby producing significantly better results over other techniques which use artificial neural networks (ANNs) to optimize with or without simulated annealing.

It has been found through study that swarms and their behavior can be applied to solve a plethora of engineering problems. Reif and Wang (1999) used a method called the "*social potential fields*" to define inverse-power force laws which dictate "*social relations*" between robots. An individual robot's motion is controlled by the resultant artificial force imposed by other robots and other components of the system. Levine and Rappel (2001) investigated a discrete model consisting of self-propelled particles that obey simple interaction rules. In this work, they demonstrated the self-organization properties of the model and the existence of coherent localized one- and two-dimensional solutions. In the one-dimensional solution, we get a constrained flock which is finitely extant with sharp drops of density down to zero at the edges. Two-dimensional vortex solutions are those in which the particles of interest rotate around a center common to all of them. Random initial conditions, even when confining boundaries are absent, can also engender the latter. Finally, Suzuki and Yamashita (1999) considered a system of multiple mobile robots in which each robot, at infinitely many unpredictable time instants, observes the positions of all the robots and moves to a new position determined by the given algorithm. The work investigates a number of formation problems by robots where they form geometric patterns in the plane where they move. Techniques of converging robots to a given point and moving a system of mobile robots to a point in a given number of finite steps are considered.

In more recent times, Cai et al. (2011) have researched on the problem of swarm stability of high-order linear-time-invariant (LTI) systems with directed graph topology. Necessary and sufficient conditions for swarm stability depending on the graph topology, the dynamics of the agents and the interaction between neighbors are derived. Ranjbar-Sahraei et al. (2012) have proposed a novel decentralized adaptive control scheme for multiagent formation control based on an integration of artificial potential functions with robust control techniques. Robust stability has been demonstrated using Lyapunov-function-based methods, which shows the robustness of the controller with respect to disturbances and system uncertainties. Dolev et al. (2013) have developed a two-phase distributed self-stabilizing scheme for producing a bounded hop-diameter communication graph. Hou and Cheah (2012) have presented a dynamic compound shape control for a swarm of robots. Each basic shape is specified by the corresponding inequality functions. With this new definition, a variety of interesting compound shapes, which are difficult to form by the existing methods, can be easily formed. A Lyapunov-like function is presented for stability analysis of the swarm systems. Guéret et al. (2012) have extended swarm computing to the Semantic Web, a system that promotes the development of the current scenario of the Web by enabling users to find and share information easily. As networking becomes more and more involved and data sets get bigger and bigger, evolutionary and swarm approaches are used to solve these pertinent problems. Yu et al. (2013) have investigated how an inversion of the swarm dynamics can help redefine the rules by which the individual agents operate in order to reach a goal of mutual interest. They have then applied this formulation to the case concerning the point defence of a very important person situated between two swarms, one of which is attacking and the other defending.

Inspired by the flocking dynamics proposed by Cucker and Smale (2007), we present a swarming dynamics and demonstrate its properties sequentially. The dynamics is found to have a conditional stability criterion, ensuring the satisfaction of which we can successfully use the model for multifarious purposes. It turns out to be capable of demonstrating converging, limit cyclic and conditionally chaotic behavior. Since we always want to avoid chaos in the system, proper tuning of the parameters will allow us to work towards this particular goal.

For the demonstration of stability and the arising of conditional chaos, we use the Lyapunov energy function construction method and the nature of the sign of the Lyapunov exponent (Cencini et al., 2010; Wolf et al., 1985). The analytical treatment is supported by a copious number of computer simulations, all of which indicate the validity of the former. Finally, we conclude by proposing a simple yet effective practical problem to which the newly proposed dynamics can be efficiently applied.