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Brief paper

# Solving the forward kinematics problem in parallel robots using Support Vector Regression

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ABSTRACT

The Stewart platform, a representative of the class of parallel manipulators, has been successfully used in a wide variety of fields and industries, from medicine to automotive. Parallel robots have key benefits over serial structures regarding stability and positioning capability. At the same time, they present challenges and open problems which need to be addressed in order to take full advantage of their utility. In this paper, we propose a new approach for solving one of these key aspects: the solution to the forward kinematics in real-time, an under-defined problem with a high-degree nonlinear formulation, using a popular machine learning method for classification and regression, the Support Vector Machines. Instead of solving a numerical problem, the proposed method involves applying Support Vector Regression to model the behavior of a platform in a given region or partition of the pose space. It consists of two phases, an off-line preprocessing step and a fast on-line evaluation phase. The experiments made have yielded a good approximation to the analytical solution, and have shown its suitability for real-time application.



## 1. Introduction

A few years after the term *robot* was coined, the first industrial parallel robot, a spray painting machine, was patented by Pollard (1940). Extensively used as manipulators in a wide variety of fields, e.g. medicine, optics, astronomy and many industries like aerospace, automotive and aviation, parallel robots have attracted the attention of many researchers in recent decades. They can be classified as closed-loop mechanisms, in contrast to serial robots, which are usually open-loop kinematics chains. Instead, they consist of two frames connected by means of active links, such as prismatic actuators, sometimes called *legs*. The advantages of this type of structure include a greater rigidity and positioning capability, good dynamic performance and high load carrying capacity, which make them very attractive in many applications and fields. Notable examples of parallel manipulators are the *Delta* robot (Clavel, 1988), the *Tricept* (Siciliano, 1999) and the Gough–Stewart platform. Presented by Gough (1956–1957) as a tyre testing machine, it is one of the most popular parallel manipulators. It gained popularity as a flight simulator (Stewart, 1965–1966) and is commonly known as the *Stewart platform*. The first design as a manipulator system was presented by McCallion and Pham (1979) as an assembly workstation. A detailed and very informative review by Dasgupta and Mruthyunjaya (2000) provides an extensive account of some relevant aspects of the Stewart platform.

In this paper we present a novel method for solving the Forward Kinematics Problem (FKP), still a relevant topic for some types of parallel manipulators, e.g., those with joint offsets. For such robots, unlike the inverse kinematics problem, the FKP lacks a closed-form mathematical solution. It is an under-defined problem with a high-degree of nonlinearity in which the solution in most cases is not unique. Although kinematics is one of the most studied topics of parallel robots, the FKP continues to gain interest, especially in terms of those methods which can solve it in real-time. This is essential for a platform characterization and its closed-loop control to know the position and orientation (*pose*) by means of the length of the linear actuators attached to the joints.

A review of the literature shows that the FKP in parallel robots has been solved in recent years using numerical as well as approximate methods and strategies. Liu et al. (1993) describe an analytical solution for the generalized configuration, i.e., 6-SPS (Spherical–Prismatic–Spherical), whereas some authors propose a simplification of the model (Nanua et al., 1990), which is solved with numerical methods like Newton–Raphson, or use of an interval analysis (Merlet, 2004) to work out the solution. However, these methods lack generalization, since they are proposed for particular types of platform. Other methods develop a solution

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to the FKP by directly applying a numerical method (Dieudonne et al., 1972) like Newton–Raphson. For instance, Dalvand and Shirinzadeh (2011, 2012) proposed an algorithm which yields solutions by means of an iterative method for a platform with joint offsets, a 6-RRCRR (Rotational–Rotational–Cylindrical–Rotational–Rotational) parallel manipulator, which does not have a closed-form solution for the FKP. In general, these techniques can achieve accurate solutions with enough iterations of the algorithms, but may have convergence problems and high computational requirements. Furthermore, there are other solutions that add rotary type sensors or extra links (Merlet, 1993; Chen and Fu, 2006) to obtain additional information, in order to aid and simplify the algorithms. However, these approaches may be difficult to generalize and in some cases will not be applicable due to structural or mechanical constraints. In addition, approximate solution strategies have been successfully applied, mainly by means of Artificial Intelligence methods like neural networks, in order to study kinematics of serial robots (Gao et al., 2010; Karlik and Aydin, 2000; Köker, 2005; Chiddarwar and Babu, 2010) and parallel robots (Parikh and Lam, 2009; Tarokh, 2007). These strategies are more suited to real-time applications and obtain good enough approximations for a wide variety of fields.

This paper presents a spatial decomposition method for obtaining accurate solutions in real-time for the FKP of Stewart platforms using a popular machine learning method, the *Support Vector Machines* (SVMs), as the regression model. Firstly, the method is applied to a 6-SPS parallel manipulator for which an analytical solution exists. In order to verify its correctness and efficiency, the yielded results are compared with the polynomial curve fitting method proposed by Tarokh (2007), and against the exact analytical solution for a generalized Stewart platform presented by Liu et al. (1993). Secondly, a similar experiment is conducted with a real parallel manipulator, the M-850 hexapod by *Physik Instrumente*.

This paper is organized as follows. Kinematics in parallel robots are discussed in Section 2. The spatial decomposition method is introduced in Section 3, followed by the description of the classification procedure in Section 4. Then, the forward kinematics modeling with SVR machines and the *on-line* evaluation procedure are detailed in Section 5. Finally, results of different experiments are discussed in Section 6, and the main conclusions are summarized in Section 7.

## 2. On kinematics of parallel robots

### 2.1. The 6-SPS general Gough–Stewart platform

The theory of serial–parallel duality, which highlights the qualitative distinctions between serial and parallel manipulators, states that in both position and velocity there is a symmetric relationship in the forward and inverse cases (Collins and Long, 1995). In contrast to the simple forward kinematics and complicated inverse kinematics of serial manipulators (requiring the solution of a system of nonlinear equations), parallel manipulators exhibit more or less straightforward inverse kinematics and a challenging solution for the forward kinematics problem.

The generalized Gough–Stewart platform is the most celebrated manipulator in the entire class of parallel robots. It has found a central status in the literature due to the fact that it exhibits the serial–parallel duality in the most prominent manner.

As shown in Fig. 1, the generalized configuration is composed of two platforms and a set of linear actuators, typically six, often called *legs*. Consider a fixed base $B$, with a coordinate frame $XYZ$ attached to it, and another coordinate frame $xyz$ fixed to the top platform $A$. Let us define the *link space* as the 6-D space consisting of the value of the length of each leg. A *link vector* represents a position in the link space as a 6-D vector $l=[l_1,l_2,\ldots,l_6]^{\top}$. Similarly, the *pose space* is the 6-D space that represents the position and orientation of the top platform (or end effector), as a combination in 3-D Cartesian coordinates of position $[x,y,z]$ and 3-D orientation angles $[\alpha,\beta,\gamma]$. A *pose* is defined by the vector $p=[x,y,z,\alpha,\beta,\gamma]^{\top}$.

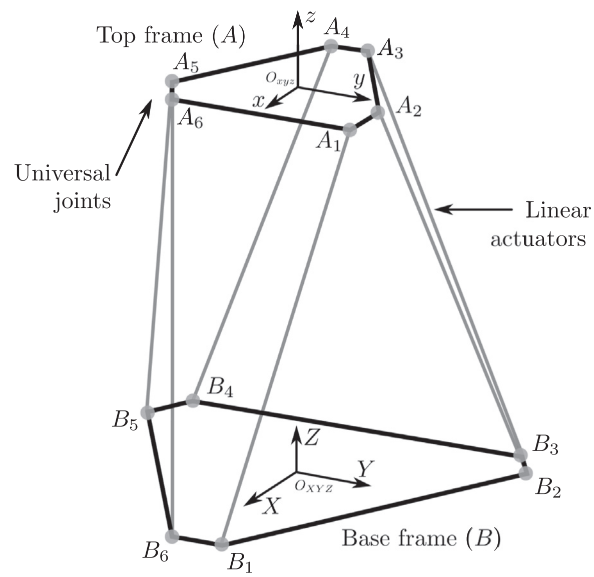

**Fig. 1.** The general Gough–Stewart platform, a class of parallel robots.

Let $d$ denote the displacement vector of the frame $[x,y,z]$ relative to the frame $XYZ$, let $B_i$ be the position of the $i$-th link (*leg*) attached to the base relative to $XYZ$, and similarly let $A_i$ be the position of the $i$-th link attached to the top platform with respect to $[x,y,z]$. Finally let $R$ be the $3\times3$ rotation matrix that defines the rotating angles of the frame $[x,y,z]$ with respect to the frame $XYZ$. Since there is no joint offsets, the length of each link connecting the base to the top can be written as

$$\begin{aligned} l_i &= \sqrt{\|RA_i+d-B_i\|} \quad i=1,2,\ldots,6 \\ &= f_i(p), \end{aligned} \tag{1}$$

where $f_i(p)$ is a known function and $p$ is the top platform pose. It is noted that $A_i$ and $B_i$ are known for a given hexapod. Furthermore, specification of the pose, i.e., position and orientation of the top platform, determines the rotation matrix $R$ and displacement vector $d$.

For parallel robots, the forward kinematics problem can be stated as follows: Given a link vector $l$, and the position of link attachments to the top and base platforms, $A_i$ and $B_i$ (with $i=1,2,3,4,5,6$), respectively, find the set of all possible poses $p$ that satisfy (1).

### 2.2. A platform with joint offsets: the 6-RRCRR case

This kind of hexapod is convenient in some situations, for example when there are manufacturing constraints, since universal joints are more complex to produce. Fig. 2 shows a diagram for a given leg. It is composed of two revolute joints, separated by an offset given by $P_J$, a prismatic joint, a passive cylindrical joint, and similarly, another two revolute joints with the same offset. For the $i$-th leg, the joint variables are $q_{ki}$, with $k=1,2,3,4,5,6$. This configuration leads to more complicated kinematics equations, since dependency between joint variables exists. An inverse kinematics solution for this parallel manipulator is the distance between joints $q_{1i}$ and $q_{6i}$, i.e., the Euclidean norm of vector $q_{3i}$,