



Surrogate relaxation of a fuzzy multidimensional 0–1 knapsack model by surrogate constraint normalization rules and a methodology for multi-attribute project portfolio selection

Esra Bas*

Department of Industrial Engineering, Istanbul Technical University, Macka 34367, Istanbul, Turkey

ARTICLE INFO

Article history:

Received 3 April 2011

Received in revised form

21 August 2011

Accepted 16 September 2011

Available online 5 October 2011

Keywords:

Fuzzy multidimensional 0–1 knapsack model

Surrogate constraint normalization rule

t-Norm/t-conorm fuzzy relations

Project selection

Fuzzy SAW

Multiple attributes

Construction

ABSTRACT

In this paper, a multidimensional 0–1 knapsack model with fuzzy parameters is defuzzified using triangular norm (t-norm) and t-conorm fuzzy relations. In the first part of the paper, the surrogate relaxation models of the defuzzified models are developed, and the use of surrogate constraint normalization rules is proposed as the surrogate multipliers. A methodology is proposed to evaluate some surrogate constraint normalization rules proposed in the literature as well as one rule proposed in this paper. Three distance metrics are used to find the distance of fuzzy objective function from the surrogate models to the distance of fuzzy objective function from the original models. A numerical experiment shows that the rule proposed in this paper dominates the other rules considered in this paper for three distance metrics given the whole assumptions. In the second part of the paper, a methodology is proposed for multi-attribute project portfolio selection, and optimal solutions from the original defuzzified models as well as near-optimal solutions from their surrogate relaxation models are considered as alternatives. The aggregation of evaluation results is managed using a simple yet effective method so-called fuzzy Simple Additive Weighting (SAW) method. Then, the methodology is applied to a hypothetical construction project portfolio selection problem with multiple attributes.

© 2011 Elsevier Ltd. All rights reserved.

1. Introduction

A deterministic multidimensional 0–1 knapsack model is already significant with a large domain of application areas (Fréville, 2004). An early application has been in capital budgeting problems. Some other applications have been cutting stock and loading problems, while the importance of a multidimensional 0–1 knapsack model as a subproblem of other integer problems has also been emphasized in the literature (Fréville, 2004). Exact methods, heuristics including greedy algorithms, and metaheuristics are some of the solution approaches to the problem (Fréville, 2004), whereas surrogate constraint relaxation is one of the methods for relaxation of a multidimensional 0–1 knapsack model as proposed by Glover (1968). Searching for optimal surrogate multipliers has been an important research area (for example see, Karwan and Rardin, 1984). Alternatively, Ablanedo-Rosas and Rego (2010) proposed a set of surrogate constraint normalization rules as surrogate multipliers for the set covering problem, and tested the strong duality gap.

Although remarkable contributions have been made in surrogate constraint relaxation research area, the uncertainty in

parameters of a surrogate relaxation of an integer programming model has been relatively neglected. Hu and Fang (2011) proposed a set-covering based surrogate approach and surrogate heuristic to solve a multidimensional 0–1 knapsack model with continuous t-norms. Several other approaches can also be proposed to address the uncertainty in parameters of a multidimensional 0–1 knapsack model and its surrogate relaxation, i.e., robust or stochastic optimization approaches can model the behavior of uncertain parameters.

In the first part of the paper, we consider a multidimensional 0–1 knapsack model with all parameters as fuzzy numbers. A fuzzy model can be defuzzified using a wide variety of fuzzy relations (for a complete overview, please see Chen and Hwang, 1992). Bas and Kahraman (2009) proposed a fuzzy capital rationing model, which is a kind of a continuous multidimensional knapsack model. Then, they defuzzified the model by t-norm fuzzy relation, analyzed it by weak duality, complementary slackness, and sensitivity analysis results. In this paper, we defuzzify a multidimensional 0–1 knapsack model with fuzzy parameters using t-norm and t-conorm fuzzy relations. Each defuzzified model gives the results for the α -cut of the fuzzy parameters. While the model as a result of defuzzification by t-conorm fuzzy relation gives the worst value of the α -cut of the fuzzy objective function, the model as a result of defuzzification by t-norm fuzzy relation gives the best value of the α -cut of the

* Tel.: +90 212 293 13 00x2671; fax: +90 212 240 72 60.

E-mail address: atace@itu.edu.tr

fuzzy objective function. After defuzzification of the model by t-norm and t-conorm fuzzy relations, we develop the surrogate constraint relaxation of the models, and propose a methodology to evaluate one surrogate constraint normalization rule proposed in this paper as well as some other rules proposed in the literature. A numerical experiment illustrates the methodology, and shows that the rule proposed in this paper dominates the other rules considered in this paper given the assumptions considered in this paper.

In the second part of the paper, we propose a methodology to evaluate investment project portfolios with multiple attributes. In this methodology, optimal solutions from defuzzified models for the selected α -cut levels as well as near-optimal solutions from their surrogate relaxation models are considered as project portfolio alternatives. Then, the project portfolio alternatives are ranked based on fuzzy SAW methodology by considering multiple attributes. As a case study, the methodology is applied to a hypothetical construction project portfolio selection problem in Turkey. The methodology shows the applicability of the theoretical results from the first part of the paper to a project selection problem. The application can be extended to other engineering problems such as loading problem and production planning problem, which can be represented as multidimensional 0–1 or continuous knapsack models with uncertain parameters, and which should be evaluated more in detail by various attributes.

2. Fuzzy knapsack models

In this section, we provide a brief overview of fuzzy knapsack models. Table 1 includes selected articles for fuzzy 0–1 knapsack models, a few fuzzy continuous knapsack models, and some special fuzzy knapsack models.

In Table 1, “number of dimensions” refers to “number of constraints”. In this paper, we consider a fuzzy multidimensional, single objective 0–1 knapsack model with fuzzy goal and all parameters as fuzzy numbers, consider α -cut representation and t-norm and t-conorm fuzzy relations for defuzzification, and develop surrogate relaxation of the defuzzified models. Then, we integrate the results from multidimensional 0–1 knapsack models with a methodology for multi-attribute project portfolio selection problem, aggregate the results by fuzzy SAW method, and apply the methodology to construction project portfolio selection as an example of engineering application. In Table 1, Chang and Lee (2012) integrated knapsack model with Data Envelopment Analysis (DEA), Bas (2011) integrated knapsack model with Analytic Hierarchy Process (AHP), and Cadenas et al. (2011) used Support Vector Machine (SVM) as a data mining technique to be integrated with knapsack model. Although the knapsack model formulation by Ishibuchi et al. (2010) is not fuzzy, during the so-called multiobjectivization, the objective function is parametrized analogous to α -cut representation in a fuzzy model.

3. Fuzzy multidimensional 0–1 knapsack model defuzzified by t-norm and t-conorm fuzzy relations

There are various ways of defuzzifying a fuzzy mathematical programming model using different fuzzy relations (for a complete overview of fuzzy relations, please see Chen and Hwang, 1992). Among these methods, α -cut representation is well-known, while Hamming distance, comparison function, probability distribution, centroid index, and area measurement are some other examples (Chen and Hwang, 1992). In this paper, we use α -cut representation, and assume fuzzy goal for the satisfaction of

objective function. Then, we use fuzzy extensions of inequality relations defuzzified by t-norm and t-conorm fuzzy relations as extensions of possibility and necessity measures classified under comparison function (Chen and Hwang, 1992; Ramik, 2006). The basic motivation for the author to use t-conorm fuzzy relation was the similar nature of the defuzzified model with robust formulation of a linear programming (LP) model originally proposed by Soyster (1973). Since after, robust LP was studied more extensively in other papers including Ben-Tal and Nemirovski (2000) and Bertsimas and Sim (2004). Soyster's (1973) model defines each parameter as a convex set, assumes worst-case scenario, and is analogous to a fuzzy model with fuzzy inequality relations defuzzified by t-conorm fuzzy relation, where the parameters are assumed to be fuzzy sets. Thus, the fuzzy model defuzzified by t-conorm fuzzy relation provides the worst-case results for the given α -cut. Alternatively, a fuzzy model with fuzzy inequality relations defuzzified by t-norm fuzzy relation provides the best-case results for the given α -cut. Chen (2009) proposed a fuzzy continuous knapsack model with a single objective function, a single constraint and fuzzy weights, considered α -cut representation, and obtained two-level mathematical programming formulation by considering Zadeh's extension principle and dual model. Their parametric programming approach is analogous to our model. However, they do not explicitly use t-norm and t-conorm fuzzy relations, and obtain their final models in two levels using dual model, whereas we obtain our model in a single level using t-norm and t-conorm fuzzy relations. Inuiguchi et al. (2003) extensively studied an LP model with fuzzy inequality relations defuzzified by t-norm and t-conorm fuzzy relations. Bas and Kahraman (2009) developed a fuzzy capital rationing model with fuzzy inequalities defuzzified by t-norm fuzzy relation, and Bas (2011) proposed a fuzzy 0–1 bidimensional knapsack model for workplace mobbing prevention investment with fuzzy inequalities defuzzified by t-norm and t-conorm fuzzy relations.

3.1. Preliminaries

The preliminaries for fuzzy set theory only relevant to the scope of the paper will be provided.

If $\tilde{A}$ is a fuzzy set with the membership function $\mu_{\tilde{A}} : X \rightarrow [0,1]$, then $\tilde{A}$ is a *normal* fuzzy set if there exists $\bar{x} \in X$ such that $\mu_{\tilde{A}}(\bar{x}) = 1$, where $\bar{x}$ is the *core* of $\tilde{A}$. $\tilde{A}$ is a *convex* fuzzy set if for all elements $x_1 < x_2 < x_3$ of fuzzy set $\tilde{A}$, $\mu_{\tilde{A}}(x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_3))$ holds (Ross, 1995; Bas and Kahraman, 2009).

An α -cut of fuzzy set $\tilde{A}$ is defined as $[\tilde{A}]_{\alpha} = \{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}$ and *strict* α -cut of fuzzy set $\tilde{A}$ is defined as $(\tilde{A})_{\alpha} = \{x \in X | \mu_{\tilde{A}}(x) > \alpha\}$ (Inuiguchi et al., 2003; Bas and Kahraman, 2009).

If $\leq$ is a crisp binary relation, $\tilde{\leq}$ is a fuzzy extension of the crisp binary relation $\leq$, $\tilde{A}$ and $\tilde{B}$ are normal and compact fuzzy sets, and $T = \min$ is a t-norm, and $S = \max$ is a t-conorm, then the following identities are equivalent (Inuiguchi et al., 2003; Bas and Kahraman, 2009):

$$\mu_{\tilde{\leq}}^T(\tilde{A}, \tilde{B}) \geq \alpha \text{ if and only if } \inf[\tilde{A}]_{\alpha} \leq \sup[\tilde{B}]_{\alpha} \quad (1)$$

$$\mu_{\tilde{\leq}}^S(\tilde{A}, \tilde{B}) \geq \alpha \text{ if and only if } \sup(\tilde{A})_{1-\alpha} \leq \inf(\tilde{B})_{1-\alpha} \quad (2)$$

With respect to strict α -cut and α -cut of a fuzzy set $\tilde{A}$, the following notations will be used throughout the paper (Inuiguchi et al., 2003; Bas and Kahraman, 2009):

$$\tilde{A}^L(\alpha) = \inf\{a \in R | a \in [\tilde{A}]_{\alpha}\}, \tilde{A}^R(\alpha) = \sup\{a \in R | a \in [\tilde{A}]_{\alpha}\} \quad (3)$$

$$\tilde{A}^L(\alpha) = \inf\{a \in R | a \in (\tilde{A})_{\alpha}\}, \tilde{A}^R(\alpha) = \sup\{a \in R | a \in (\tilde{A})_{\alpha}\} \quad (4)$$

respectively. When $\tilde{A}$ is a strictly convex and normal fuzzy number, then the following also holds true (Inuiguchi et al.,

Download English Version:

<https://daneshyari.com/en/article/381023>

Download Persian Version:

<https://daneshyari.com/article/381023>

[Daneshyari.com](https://daneshyari.com)