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Inverse propagation of uncertainties in finite element model updating through use of fuzzy arithmetic

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ABSTRACT

A fuzzy finite element model updating (FFEMU) method is presented in this study for the damage detection problem. The uncertainty caused by the measurement noise in modal parameters is described by fuzzy numbers. Inverse analysis is formulated as a constrained optimization problem at each α-cut level. Membership functions of each updating parameter which correspond to reduction in bending stiffness of the finite elements is determined by minimizing an objective function using a hybrid version of genetic algorithms (GA) and particle swarm optimization method (PSO) which is very efficient in terms of accuracy and robustness. Practical evaluation of the approximate bounds of the interval modal parameters in FFEMU iterations is addressed. A probabilistic analysis is performed using Monte Carlo simulation (MCS) and the results are compared with presented FFEMU method. It is apparent from numerical simulations that the proposed method is well capable in finding the membership functions of the updating parameters within reasonable accuracy. It is also shown that the results obtained by FFEMU are in good agreement with the MCS results while FFEMU is not as computationally expensive as the MCS method. Nevertheless, the proposed FFEMU do not required derivatives of the objective function like existing methods except in the deterministic case.

1. Introduction

Presence of uncertainties is unavoidable in the real life engineering and mechanical systems. Thus, uncertainty quantification gains considerable attention from many scientific disciplines. In the context of structural engineering, there may be uncertainties associated with the model parameters or the model itself due to mathematical simplifications which have remarkable effect on system responses (e.g., material properties, connection rigidities, soil parameters). Nevertheless, measurement noises are inevitably involved in the experimental studies. In order to make reliable estimation for the system output and assess the system reliability, some uncertainty quantification techniques such as Monte Carlo simulation, mean centered perturbation method, interval arithmetic and fuzzy sets are proposed to quantify uncertainties in engineering problems. Detailed discussion on these methods can be found in Khodaparast (2010), Marwala (2010), Henriques (2008).

Uncertainty quantification methods can be separated into two groups: Probabilistic methods and non-probabilistic methods. Probabilistic methods are based on the probability theory in which the uncertain parameters are considered as random parameters. The frequency of occurrences of these random parameters is given by probability density functions. The MSC, perturbation methods and the asymptotic integral method can be classified as probabilistic methods. However, the interval arithmetic and the fuzzy set theory are in the class of non-probabilistic methods since no assumption is made about the probability distribution of uncertain variables.

Various methods are proposed for forward uncertainty propagation in engineering systems using probabilistic and non-probabilistic methods. Adhikari and Friswell M I (2007) proposed two perturbation based methods in order to evaluate the statistics of modal parameters of structural systems. Degrauwe et al. (2010) presented an interval analysis method improved by affine arithmetic to reduce the overestimation of uncertainty on the results due to dependency problem. A finite element method for fuzzy structures is proposed by Zhenyu and Qiu (2002). They transferred the fuzzy variable into random variables using the information entropy and used the perturbation finite element method to calculate the statistics of structural response. Some alternative fuzzy arithmetic and fuzzy finite element methods are presented by many authors in order to tackle with both epistemic and irreducible uncertainty (Balu and Rao, 2012; Nicolai et al., 2011; Massa et al., 2008; Huang and Li, 2005; Chen and Rao, 1997). However, there

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are limited numbers of studies where the uncertainty propagation techniques are applied to inverse problems such as the Finite Element Model Updating (FEMU).

The FEMU is a very attractive technique to assess the structural integrity. The goal of the FEMU is to identify some imprecise model parameters such as material properties, boundary conditions and beam column connections by minimizing an objective function which involves the differences between the experimental and numerical modal parameters. The damage in civil engineering structures can also be identified by the FEMU with the assumption that the structural damage usually causes a decrease in structural stiffness (Yu and Chung, 2012, Teughels, 2007; Bakir et al., 2007, 2008). However, there are always errors in the measured data and the numerical model that affect the results of the FEMU (Frisswell, 2007). In order to identify the propagation of the uncertainties in updating parameters through FEMU, probabilistic and non-probabilistic methods can be used. The distributions or the range of updating parameters in the parameter space can be estimated from the measured variability of the output data such as modal parameters. This is called the stochastic finite element modal updating. Hua et al. (2008) presented a perturbation based stochastic finite element model updating method in which they use a gradient based iterative method in order to determine the mean values and the coefficient of variations of the updated parameters due to the uncertainties in material properties and measurement data. They compare the results considering the correlation between updating parameters and the modal parameter with Monte Carlo simulation (MCS). Khodaparast et al. (2008) adapted a similar method and proposed two perturbation based methods in order to determine the first and the second statistical moments of updating parameters. They investigate the influence of correlation between the updated and the modal parameters on the results of a 3 degree-of-freedom simple system and truss structure. Moaveni and Conte (2009) presented a study in order to quantify and analyze uncertainties in damage detection problem using finite element model updating strategy. The aim of their study was to investigate the influence of the uncertainties in modal parameters, spatial density of measurement and the mesh size of the FE model in the damage detection results. They perform a gradient based optimization method to minimize the objective function which includes the residuals of experimental and numerical modal parameters for different combinations of uncertainty source. An interval based model updating strategy with irreducible uncertainty is proposed by Khodaparast et al. (2011). The Kriging predictor was chosen as meta-model in order to estimate the bounds of output parameters efficiently. Govers and Link (2010) used statistical properties of multiple sets of experimental data for stochastic FEMU. They presented a method to adjust the design parameter means and related covariance matrix using the extended version of classical gradient based iteration. A fuzzy logic system is developed by Chandrashekhar and Ganguli (2009) to identify the damage in a cantilever beam in the presence of measurement noise. The connection between probability theory and the fuzzy logic was also addressed in their studies.

In the present study, a fuzzy finite element model updating methodology is presented in order to examine the uncertainty propagation in the updating parameters. General framework is defined to make FFEMU procedure applicable to large-scale engineering problems. The inverse FFEMU problem is considered as an optimization problem. Uncertainties in modal parameters are represented by the fuzzy numbers and then fuzzy arithmetic operations are carried out in interval arithmetic sense using α-cut procedure. Practical calculation of interval bounds of the response parameters inside the optimization loop using Taylor series expansion which facilitate the fuzzy updating are explained in detail. An objective function is constructed including the interval bounds of the experimental and numerical modal parameters. An efficient hybrid version of GA and PSO is proposed and used to obtain the global optimum of the given objective function at each α-cut level. By this way, nonlinearity between input and out parameters is captured. Calculation of interval modal parameters would be very challenging task in some cases especially when the input–output relation is not monotonic. In this study, it is shown that two FE calculations are needed to find the lower and upper bounds for each response parameter by using the information obtained from a preliminary analysis in the input parameter domain. Finally, the proposed FFEMU procedure is compared with the MCS results. A three storey and two bay frame structure is used to verify the results obtained by FFEMU. The results show that the updated uncertain parameters obtained by FFEMU are found to be in good agreement with MCS results. Nevertheless, FFEMU requires less computational time while the MCS is computationally expensive and may be impractical for large numerical models. It should also be noted that this method can be applied any engineering problems in order to quantify input uncertainties from output uncertainties as long as the input–output relation can be identified by an preliminary analysis in the parameter space.

2. Fuzzy sets, numbers and arithmetic operations

The fuzzy set theory introduced by Zadeh (1965) has gained an increasing interest in many engineering fields. The fuzzy set theory is a very powerful method which is very capable in quantifying uncertainties in engineering problems. The method describes linguistic uncertainty, incomplete or imprecise information and vagueness in a non-probabilistic manner. A fuzzy set is given in Eq.(1).

$$\tilde{A} = \left(x,\mu_{\tilde{A}}(x)\right) \big| x \epsilon X, \mu_{\tilde{A}}(x) \in [0,1] \tag{1}$$

in Eq.(1), $\mu_{\tilde{A}}(x)$ is the membership function of the fuzzy set $\tilde{A}$ which expresses the degree to which a sample x belongs to the set $\tilde{A}$. A fuzzy number is a fuzzy set which satisfy the following conditions. (1) $\tilde{A}$ is normal, (2) $\tilde{A}$ is convex, (3) There is exactly one $x \in R$ with $\mu_{\tilde{A}}(x) = 1$, (4) The membership function $\mu_{\tilde{A}}(x)$, $x \epsilon R$, is at least piecewise continuous. Actually, there is infinite number of possible sets that satisfy these conditions. However, triangular, trapezoidal, and Gaussian type fuzzy sets are the prevalent fuzzy numbers in applied fuzzy arithmetic.

The α-cut of the fuzzy set $\tilde{A}$ is given in Eq. (2). The α-cut procedure is frequently used in applied fuzzy arithmetic. Some other basic definitions and notations for fuzzy sets and numbers can be found in Hanss (2005).

$$A_\alpha = x \,\epsilon\, X \mid \mu_{\tilde{A}}(x) \geq \alpha \tag{2}$$

The goal of the fuzzy arithmetic is to determine the membership functions of the system outputs on the basis of given membership functions of the system inputs. Zadeh (1965) proposed the well-known extension principle in order perform fuzzy arithmetical operations. The extension principle relates the possibility distribution of n fuzzy input variables to the possibility distribution of the output variable by Eq. (3). However, the extension principle requires extensive computational effort as the number of input variable increases. The required function evaluations in this case is d^n, where d is discrete values for each fuzzy number and n is the number of variables.

$$\mu_{\tilde{B}}(y) = \sup_{y=f(x_i)} \min \mu_{\tilde{A}_i}(x_i) \quad i = 1,2,\ldots,n \;\; \forall x_i \epsilon \Re \tag{3}$$

Another method for fuzzy arithmetic operations is the transformation method (Hanss 2005). The method is applicable for the analysis of systems with uncertain model parameters. Transformation method