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Stabilization for a class of nonlinear systems: A fuzzy logic approach ☆

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ABSTRACT

In this paper, the problem of stabilization for the class of continuous time nonlinear systems which are discretized in closed form is addressed. By using the Takagi–Sugeno model approach, a discrete controller capable of stabilizing the discrete Takagi–Sugeno model and the continuous model as well, is obtained. This scheme allows using a digital controller for stabilizing an analog plant.

1. Introduction

The stabilization problem of dynamical systems constitutes an interesting problem in control theory. Many approaches have been proposed, and continue to appear, to tackle this problem for different kinds of systems—linear or nonlinear, continuous or discrete. In general, the control law has the same nature of the system, namely, for continuous systems, continuous controllers are designed to guarantee the stability of the closed-loop system.

The use of faster digital computers has motivated the design of sampled-data controllers for continuous time plants. Depending on the holders—usually being zero order—the dynamics of the sampled nonlinear system, or discretized system for short, are usually only an approximation (Monaco and Normand-Cyrot, 1985, 1987, 1988, 2007; Nesic and Teel, 2004). Furthermore, once the (approximated) discretized system dynamics have been obtained, and the digital controller has been designed fulfilling certain design requirements, the controller performance may not be necessarily satisfactory when applied to the continuous system. This is due to the fact that the control signals differ from those of a continuous controller designed for the continuous system with the same requirements, because of the sampling and holding operations.

Several methods for approximate discretization can be found in the literature. Obviously, the performance of a controller designed on the basis of the approximate discretization depends on the degree of approximation. For example, when using the simple Euler method, it is possible that the controller does not guarantee the stability of the closed loop system (Monaco and Normand-Cyrot, 1997, 2001, 2007). A way to overcome this situation would be the determination of a discretization in closed form. Clearly, this might not be always possible. Some methods for the discretization in closed form have appeared recently in the literature and, moreover, some results point out that it is possible to induce the "discretizability property in closed form" by a suitable feedback (Di Giamberardino et al., 2000). An example of such systems are the systems completely linearizable by feedback or the class of systems transformable to a polynomial triangular form.

On the other hand, recent results on fuzzy modeling, in particular the Takagi–Sugeno (TS) modelization approach, can be fruitfully used in the problem of stabilization of nonlinear systems (Takagi and Sugeno, 1985). The main feature of the TS models is that they represent in a certain region the local dynamics of the system, by means of a linear model. The complete fuzzy model is then obtained by a fuzzy aggregation of these linear models. In this way, it can be shown that the TS fuzzy models are universal approximators of many nonlinear systems, and the design of the controller can be made on the basis of the linear subsystems describing locally the aggregate nonlinear TS model (Tanaka and Wang, 2001).

In this paper, supposing to be able to determine the discretized dynamics in closed form of a continuous time system, we

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determine a TS discrete time model. This TS model is then used to design a discrete time controller, stabilizing the TS discrete time model. Moreover, we show that this discrete controller, when implemented via a zero order holder and under certain conditions, stabilizes the continuous time system as well. The main advantage of the proposed technique relies in the simplicity of the controller design. In fact, the discrete time controller results from the fuzzy aggregation of the digital controllers, each designed for a discrete time linear system of the TS model.

2. Some facts about the discretization of dynamical systems

Consider a linear time invariant system described by

$$\dot{x} = A_c x + B_c u, \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$. It is well known that the discretization of this linear system with a sampling interval δ is given by

$$x_{k+1} = A_d x_k + B_d u_k$$

where

$$x_{k+1} = x(k\delta+\delta), \quad x_k = x(k\delta), \quad u_k = u(k\delta),$$

$$A_d = e^{A_c\delta}, \quad B_d = \int_0^\delta e^{A_c s}\, ds\, B_c.$$

For nonlinear systems, however, finding the solution of the differential equations is difficult in many cases. Hence, various authors consider approximate discretizations. As a result, at the sampling instants the solutions of the differential and approximate discretized systems do not coincide, and poor accuracy may result. Also, relatively large sampling period may cause instability or undesired behavior.

However, some cases of discretization in closed form of nonlinear systems have been studied in the literature (Monaco et al., 1996), (Di Giamberardino et al., 2006), (see also Monaco and Normand-Cyrot, 2007, and references therein). These schemes allow expressing the discretization process as a power Lie series, and discretization in closed form can be obtained if some condition on the residuals holds. To precise these ideas, let us consider the nonlinear system

$$\dot{x} = f(x,u).$$

Expanding its solutions $x(t)$ around $t=0$ we get

$$\begin{aligned} x(t) &= \left[\frac{x(t)}{0!}\right]_{t=0} + \left[\frac{\dot{x}(t)}{1!}\right]_{t=0} t + \left[\frac{\ddot{x}(t)}{2!}\right]_{t=0} t^2 + \cdots \\ &= x(0) + f(x(0),u(0))t + \frac{1}{2!}\left[\dot{f}(x,u)\right]_{t=0} t^2 + \cdots \\ &= x(0) + \sum_{i=1}^{\infty} \frac{t^i}{i!}\left[f^{(i)}(x,u,\dots u,\dots,u^{(i-1)})\right]_{t=0}, \end{aligned} \tag{2}$$

where the operator $f^{(i)}(x,u,\dot{u},\dots,u^{(i-1)})$ is defined as

$$f^{(1)}(x,u) = f(x,u),$$

$$\begin{aligned} f^{(i)}(x,u,\dots,u^{(i-1)}) &= \frac{\partial f^{(i-1)}(x,u,\dot{u},\dots,u^{(i-2)})}{\partial x} f(x,u) \\ &+ \frac{\partial f^{(i-1)}(x,u,\dot{u},\dots,u^{(i-1)})}{\partial u}\dot{u} + \cdots \\ &+ \frac{\partial f^{(i-1)}(x,u,\dot{u},\dots,u^{(i-1)})}{\partial u^{(i-2)}} u^{(i-1)}. \end{aligned}$$

Taking the solution (2) around $t = k\delta$ and considering a piecewise constant input u_k for $k\delta \le t < (k+1)\delta$, we can write the discrete solution as

$$x_{k+1} = \sum_{i=0}^{\infty} \frac{\delta^i}{i!}\left[L^i_{f(x,u)}(x)\right]_{\substack{x=x_k\\u=u_k}} = e^{\delta L^i_{f(x,u)}}(x)\big|_{\substack{x=x_k\\u=u_k}}, \tag{3}$$

where $L^i_{f(x,u)}(\cdot)$ is defined as

$$L^i_{f(x,u)}(x) = \frac{\partial L^{i-1}_{f(x,u)}}{\partial x} f(x,u), \quad L^0_{f(x,u)}(x) = x.$$

From the previous expression, if for a finite i the term $L^i_{f(x,u)}$ is zeroed, namely the nilpotency condition is fulfilled, and the discretization becomes in closed form. Otherwise, only an approximation up to a certain degree can be obtained. The condition of nilpotency is a sufficient condition for discretization in closed form.

3. The Takagi–Sugeno fuzzy model

Let us consider a continuous time nonlinear system described by

$$\dot{x} = f(x,u), \tag{4}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$. It is well known that it is possible to describe, at least in a certain region of interest, the behavior of the nonlinear system (4) by a suitable aggregation of local linear subsystems. One of these approaches is the Takagi–Sugeno modelization (Takagi and Sugeno, 1985). This model is described by a set of fuzzy IF–THEN rules representing local linear input–output dynamics of a nonlinear system. The most interesting feature in the TS model is that the local dynamics can be described by linear submodels and the aggregated model is obtained by fuzzy blending of the linear submodels. These linear models can be derivated directly from the nonlinear model by a sector nonlinearity approach, in which case the aggregated model describes exactly the nonlinear dynamics in a regions $\mathcal{D}$ of interest, or in an approximate way. In the later case, the number of the IF–THEN rules may be reduced, but the controller calculated on the basis of the approximated TS model cannot guarantee a priori the stability of the original system. A robust scheme could be a way of overcome this situation. In this work, the sector nonlinearity approach will be taken, namely, we will assume that the TS model obtained is exact with respect to the original nonlinear dynamics in a region $\mathcal{D}$ (Tanaka and Wang, 2001).

To be more precise, let us have the local subsystems defined as follows:

Plant rule i: IF z_j is $F_{ji}, j = 1,\dots,p$

THEN Σ: $\dot{x} = A_i x + B_i u, \quad i = 1,\dots,r$

where $z_1,\dots,z_p$ are measurable premise variables, which may coincide with the state vector or with a partial set of this vector through the output signals y_i. Moreover, F_{1i} are the corresponding fuzzy sets. Usually, these linear subsystems are obtained from some knowledge of the process dynamics or by their linearization about some point of interest.

For a given pair $(x(\cdot),u(\cdot))$, the aggregate fuzzy model is obtained by using a singleton fuzzifier, a product inference and a center of gravity defuzzifier, giving a Continuous Fuzzy Model (*CFM*) described by

$$\dot{x} = \sum_{i=1}^{r} \mu_i(z) A_i x + \sum_{i=1}^{r} \mu_i(z) B_i u \tag{5}$$

with $z = (z_1 \ \cdots \ z_p)^T$, where $\mu_i(z)$ is the normalized weight for each rule calculated from the membership functions for z_j in F_{ji}, and such that $\mu_i(z) \ge 0$, and

$$\sum_{i=1}^{r} \mu_i(z) = 1.$$

The sampled version of system (4), using zero order holders, is given by

$$x_{k+1} = f(x_k, u_k). \tag{6}$$