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Explicit output-feedback nonlinear predictive control based on black-box models ☆

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ABSTRACT

Nonlinear model predictive control (NMPC) algorithms are based on various nonlinear models. A number of on-line optimization approaches for output-feedback NMPC based on various black-box models can be found in the literature. However, NMPC involving on-line optimization is computationally very demanding. On the other hand, an explicit solution to the NMPC problem would allow efficient on-line computations as well as verifiability of the implementation. This paper applies an approximate multi-parametric nonlinear programming approach to explicitly solve output-feedback NMPC problems for constrained nonlinear systems described by black-box models. In particular, neural network models are used and the optimal regulation problem is considered. A dual-mode control strategy is employed in order to achieve an offset-free closed-loop response in the presence of bounded disturbances and/or model errors. The approach is applied to design an explicit NMPC for regulation of a pH maintaining system. The verification of the NMPC controller performance is based on simulation experiments.

1. Introduction

Nonlinear model predictive control (NMPC) involves the solution at each sampling instant of a finite horizon optimal control problem subject to nonlinear system dynamics and state and input constraints (Mayne and Michalska, 1990; Allgöwer and Zheng, 2000; Kouvaritakis and Cannon, 2001). A survey of the numerical methods for on-line solution of NMPC problems is given in Diehl et al. (2009). Most recently, an advanced-step NMPC controller with reduced on-line computational costs has been proposed in Zavala and Biegler (2009). The NMPC algorithms are based on various nonlinear models. Often these models are developed as first-principles models, but other approaches, like black-box identification approaches are also popular. In this paper we focus on explicit solution of output-feedback NMPC problems based on black-box models.

There exists a number of NMPC approaches based on various black-box models e.g. based on neural network models (e.g. Nørgaard et al., 2000; Zeng et al., 2003), fuzzy models (e.g. Lepetič et al., 2003), local model networks (e.g. Peng et al., 2007), Gaussian Process models (e.g. Kocijan and Murray-Smith, 2005). The common feature of these NMPC approaches is that an on-line optimization needs to be performed in order to compute the optimal control input. Consequently, the computation is time consuming and the real-time NMPC implementation is limited to processes where the sampling time is sufficient to support the computational needs. However, the on-line computational complexity can be circumvented with an explicit approach to NMPC, where the only computation performed on-line would be a simple function evaluation.

It has been shown that the explicit solution to linear constrained MPC problems has an explicit representation as a piece-wise linear (PWL) state feedback law defined on a polyhedral partition of the state space (Bemporad et al., 2002). The benefits of an explicit solution, in addition to the efficient on-line computations, include also verifiability of the implementation, which is an essential issue in safety-critical applications. In Alessio and Bemporad (2009), the main contributions on explicit MPC, which have appeared in the scientific literature, are reviewed. For nonlinear MPC, the prospects of explicit solutions are even higher than for linear MPC, since the benefits of computational efficiency and verifiability are even more important. Recently, several approaches to explicit solution of NMPC problems have been suggested. An approach for efficient on-line computation of NMPC for constrained input-affine nonlinear systems has been suggested in Bacic et al. (2003). In Johansen (2002, 2004) and Grancharova et al. (2007a), approaches for off-line computation of explicit sub-optimal PWL predictive controllers for

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general nonlinear systems with state and input constraints have been developed, based on the multi-parametric nonlinear programming (mp-NLP) ideas (Fiacco, 1983). It has been shown that for convex mp-NLP problems, it is straightforward to impose tolerances on the level of approximation such that theoretical properties like asymptotic stability of the sub-optimal feedback controller can be ensured (Johansen, 2004; Bemporad and Filippi, 2006). In Grancharova et al. (2007a), practical computational methods to handle non-convex mp-NLP problems have been suggested that not necessarily lead to guaranteed properties, but when combined with verification and analysis methods give a practical tool for development and implementation of explicit NMPC. Algorithms for solving mp-NLP problems, including the non-convex case, are described also in Pistikopoulos et al. (2007). It should be noted that the mentioned methods for explicit NMPC are based on first-principles models of the systems and they assume that the state variables can be measured. Further, in Grancharova et al. (2007b), an approach for off-line computation of explicit *stochastic* NMPC controller for constrained nonlinear systems based on a stochastic black-box model (Gaussian process model) has been proposed. In addition to the mentioned methods, there exists another group of approaches for off-line computation of sub-optimal controllers, where the optimal solution is approximated by means of neural networks (Parisini and Zoppoli, 1995; Parisini and Sacone, 2001; Bertsekas and Tsitsiklis, 1998; Åkesson and Toivonen, 2006).

This paper suggests an approximate mp-NLP approach to explicit solution of *deterministic* NMPC problems for constrained nonlinear systems described by black-box models (NARX models (Chen and Billings, 1989)). In particular, neural network NARX models are considered (Chen et al., 1990). The approach builds an orthogonal search tree structure of the *regressor* space partition and consists in constructing a PWL approximation to the optimal control sequence by applying the approximate mp-NLP algorithm in Grancharova et al. (2007a). A dual-mode control strategy is proposed in order to achieve an offset-free closed-loop response in the presence of bounded disturbances and/or model errors. It is similar to the dual-mode receding horizon control concept developed in Michalska and Mayne (1993) (based on state space models), however, here black-box models are considered and an explicit solution of the NMPC problem is sought. Thus, the suggested strategy consists in using the explicit NMPC (based on NARX model) when the output variable is far from the origin and applying an LQR in a neighborhood of the origin. The LQR design is based on an augmented linear ARX model which takes into account the integral regulation error. The main motivations behind the dual-mode control strategy are the following. First, it may be beneficial to use a separate linear model in a neighborhood of the equilibrium, since the nonlinear black-box model may not have accurate linearizations unlike a first-principles model, and the requirement for accurate control is highest at the equilibrium. Second, it leads to a reduced complexity of the explicit NMPC compared to augmenting the nonlinear model with an integrator to achieve an integral action directly in the NMPC.

The following abbreviation and notation will be used in the paper. The nonlinear model predictive control problem based on black-box model will be referred to as BB-NMPC problem. $A \succ 0$ means that the square matrix A is positive definite. For $x \in \mathbb{R}^n$, the Euclidean norm is $\|x\| = \sqrt{x^T x}$ and the weighted norm is defined for some symmetric matrix $A \succ 0$ as $\|x\|_A = \sqrt{x^T A x}$.

2. Formulation of the BB-NMPC problem as an mp-NLP problem

2.1. Modelling of dynamic systems with NARX models

The black-box identification of nonlinear systems is an area which is quite diverse. It covers topics from mathematical approximation theory, estimation theory, non-parametric regression and concepts like neural networks, fuzzy models, wavelets etc. A unified overview of this topic is given in Sjöberg et al. (1995).

Consider a nonlinear dynamical system with input $u \in \mathbb{R}^m$ and output $y \in \mathbb{R}^p$ and let $U = [u(1),u(2),\ldots,u(M)]$ and $Y = [y(1),y(2),\ldots,y(M)]$ be sets of observed values of u and y to the number of M. Based on these data, the dynamics of the system can be described with a NARX model, where the future predicted output $y(i+1)$ depends on previous estimated outputs, as well as on previous control inputs:

$$y(i+1) = f(z(i),\theta), \tag{1}$$

$$z(i) = [y(i),y(i-1),\ldots,y(i-L),u(i),u(i-1),\ldots,u(i-L)]. \tag{2}$$

Here, L is a given lag, i denotes the consecutive index of data samples ($i \geq L$), $z(i)$ is the so called *regressor* vector, f is the function realized by the black-box model, and θ is a finite-dimensional vector of parameters. Thus, the function f is a concatenation of two mappings: one that takes the increasing number of the past values of the observed inputs and outputs and maps them into the finite dimensional *regressor* vector and one that takes this vector to the space of the outputs. The nonlinear mapping from the *regressor* space to the output space can be of various kinds. In our case we will use neural network with sigmoid basis functions in the hidden layer and linear basis functions in the output layer. This form of neural network is called multilayer perceptron (MLP), which is probably the most frequently considered member of the neural network family (e.g. Nørgaard et al., 2000) and can be used as an universal approximator. This particular choice was subjective. Any other choice of *regressor* vector composition or any other choice of mapping is possible until it enables satisfactory description of the modelled dynamic system. The results given in the continuation of the paper are not limited to MLP approach only.

The parameters of the MLP are the weights of its units. After the structure (number of layers and units) is determined, the model parameters are obtained with optimization, based on a chosen cost function. This cost function is most frequently a least squares combination of errors between estimated and measured output signals:

$$E = \frac{1}{2M}\sum_{i=1}^{M} \|y(i) - \hat{y}(i|\theta)\|^2, \tag{3}$$

where $\hat{y}(i|\theta)$ is estimated output signal, θ is a vector containing the weights, and M is the number of measured output signals $y(i)$. The quality of prediction can be assessed with evaluation of residuals, estimation of the average prediction error or visualization of the network model's ability to predict. The reader is referred to Nørgaard et al. (2000) for more details.

2.2. Formulation of the BB-NMPC problem

Consider the discrete-time nonlinear system:

$$x(t+1) = h(x(t),u(t)), \tag{4}$$

$$y(t) = g(x(t),u(t)), \tag{5}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^p$ are the state, input and output vectors, and $h : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$ are nonlinear functions. The following input and output constraints are imposed on the system (4)–(5):

$$u_{\min} \leq u(t) \leq u_{\max}, \quad y_{\min} \leq y(t) \leq y_{\max}. \tag{6}$$

Assume that the dynamics of the nonlinear system (4)–(5) is approximated with an MLP neural network with NARX structure of the form (1)–(2). Then for $t \geq L$, define a *modified regressor* vector:

$$\tilde{z}(t) = \begin{cases} [y(t),y(t-1),\ldots,y(t-L), \\ \qquad u(t-1),\ldots,u(t-L)] & \text{if } L > 0, \\ y(t) & \text{if } L = 0, \end{cases} \tag{7}$$