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Fractional differentiation and non-Pareto multiobjective optimization for image thresholding

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ABSTRACT

Various techniques have previously been proposed for single-stage thresholding of images to separate objects from the background. Although these global or local thresholding techniques have proven effective on particular types of images, none of them is able to produce consistently good results on a wide range of existing images. Here, a new image histogram thresholding method, called TDFD, based on digital fractional differentiation is presented for gray-level image thresholding. The proposed method exploits the properties of the digital fractional differentiation and is based on the assumption that the pixel appearance probabilities in the image are related. To select the best fractional differentiation order that corresponds to the best threshold, a new algorithm based on non-Pareto multiobjective optimization is presented. A new geometric regularity criterion is also proposed to select the best thresholded image. In order to illustrate the efficiency of our method, a comparison was performed with five competing methods: the Otsu method, the Kapur method, EM algorithm based method, valley emphasis method, and two-dimensional Tsallis entropy based method. With respect to the mode of visualization, object size and image contrast, the experimental results show that the segmentation method based on fractional differentiation is more robust than the other methods.

1. Introduction

Thresholding is a technique of image segmentation that can be effectively applied to many different types of images. It is particularly useful in cases where it simplifies the image content to such an extent that a decision can be made without further processing. Because of its low cost in CPU time and simplicity in implementation, thresholding has been the most popular approach to image segmentation and has been extensively reviewed in the literature. Surveys of image thresholding techniques were presented by Weszka (1978), Sahoo et al. (1988), and more recently by Sezgin et al. (2004). There are a lot of approaches to classify thresholding methods; authors in Sezgin et al. (2004) labelled the methods according to the information they exploit, such as histogram shape, space measurement clustering, entropy, object attributes, spatial information and local gray-level surface. Another classification approach consists in dividing these techniques into parametric and non-parametric techniques. Compared to the non-parametric techniques, the parametric techniques are less efficient. The parametric thresholding methods exploit the first-order statistical characterization of the image to be segmented. Weszka et al. (1979) proposed a parametric method where the gray-level distribution of each class was assumed to be a Gaussian distribution. The principle of this method consists in applying an optimization technique to estimate the set of Gaussian parameters that enable the best histogram fit. The optimal threshold is calculated by minimizing the overall probability error between these Gaussian distributions. The drawbacks of the parametric methods are: (i) the optimal thresholds are not always located at the intersections of the Gaussians; (ii) their effectiveness is strongly reduced when image histogram is unimodal or when the two classes overlap significantly; (iii) in most cases, the distributions of the different image classes are far from being Gaussian; (iv) execution time becomes prohibitive when the number of classes increases.

Many authors tried to solve these problems; Synder et al. (1990) proposed to fit the image histogram by means of a heuristic method. Genetic algorithms were also used to solve this problem (Yin, 1999; Bazi et al. 2007). Recently, Zahara et al. (2004) proposed a hybrid optimization technique based on particle swarm optimization to reduce the execution time. However, they did not solve the problem of the algorithm initialization and proof of the algorithm's convergence towards the optimal threshold was not given.

Whereas the non-parametric methods try to separate two successive gray-level classes by optimizing some *a posteriori* criterion, without estimating the parameters of the two distributions. Otsu (1979) and others (Ng, 2006) used the between-class

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variance criterion; the entropy measure was introduced by Kapur et al. (1985). Li et al. (1993) presented a result obtained through the use of cross-entropy and more recently Portes de Albuquerque et al. (2004) used non-extensive entropy, also called the Tsallis entropy criterion. Abutaleb (1989) and Feng et al. (2005) extended the use of the entropy to the two-dimensional case; they maximized the 2D-entropy computed from the 2D-histogram of the original image. Other papers such as Beghdadi et al. (1995), Brink (1992), Cheng and Chen (1999), Sahoo and Arora, (2006) described some criteria derived from 2D-entropy.

The theory of non-integer (fractional) order derivatives dates back to correspondence between Leibniz and L'Hospital in 1695 (see Oldham and Spanier, 1974). The basis for defining fractional derivatives is the relationship between the integer n and the nth order derivatives. A remarkable merit of fractional differentiation operators is that they may still be applied to functions which are not differentiable in the classical sense. Unlike the integer order derivative, the fractional order derivative at point x is not determined by an arbitrary small neighborhood of x. In other words, the fractional derivative is not a local property of the function. There exist several well-known approaches to unification of differentiation and integration notions, and their extension to non-integer orders (see Prodlubny, 2002). A general survey on the different approaches is given in Miller and Ross (1993).

The theory of fractional derivatives was primarily developed as a theoretical field of mathematics. More recently, fractional differentiation has found applications in various areas: in control theory, it is used to determinate a robust command control (Oustaloup and Linarès, 1996); it is also used to solve the inverse heat conduction problem (Battaglia et al. (2001); other applications are reported for instance in neuronal modelling Ramus-Serment et al. (2002), in image processing for edge detection (Mathieu et al., 2003), and in biomedical signal processing (Ferdi et al. 2000).

This paper shows how using fractional differentiation with an optimal order can provide an optimal threshold for gray-level image segmentation. A detailed interpretation and a statistical analysis of the fractional differentiated histogram are proposed in terms of amplitude range variations and autocorrelation function. To select the best thresholded image, a selection operator is also proposed, based on a new geometric regularity criterion. The computational complexity of the digital fractional differentiation is also presented. This paper finally evaluates the improvement of the proposed method over five other competing methods.

The rest of the paper is organized as follows. In Section 2, the formalism of fractional differentiation calculus is given. In Section 3, we present the properties of the fractional differentiated image histogram. In Section 4, we explain and analyze our thresholding algorithm. Some experimental results are shown in Section 5. Finally, we conclude in the last section.

2. Formalism of fractional differentiation

The Riemann–Liouville operator, for fractional differentiation, is defined by the formula (Oldham and Spanier, 1974)

$$D^{-a}f(x) = \frac{1}{\Gamma(\alpha)}\int_c^x (x-\xi)^{\alpha-1} f(\xi)\,\mathrm{d}\xi \tag{1}$$

with $f(x)$ a real causal function, $x>0$, α the fractional integration order, $\mathrm{Re}(\alpha) > 0$ (it can be any complex or real number), c the integral reference and Γ the Euler-gamma function. When the real part of the fractional order (α) is negative, a fractional integral operator is in fact also defined by (1).

When c is equal to 0, the approximation of the discrete form of fractional differentiation (DFD) of order α is then given by (Oldham and Spanier, 1974)

$$g(x) = D^{\alpha}f(x) \approx \frac{1}{h^{\alpha}}\sum_{k=0}^{M}\omega_k(\alpha)f(x-kh) \tag{2}$$

where h is the sampling step, M is the number of samples, $x = Mh$, and the coefficients $\omega_k(\alpha)$ are defined by

$$\omega_0(\alpha) = 1,\ \omega_{k+1}(\alpha) = \frac{(k+1)-\alpha-1}{(k+1)}\omega_k(\alpha), \quad k = 0, 1, 2, \ldots, M-1 \tag{3}$$

The expression (2) becomes the same as Riemann–Liouville fractional integral when h tends toward zero. From the expression (2), the function $g(x)$ can be interpreted as the output function of a discrete filter, the input of which is $f(x)$. Its impulse response is then given by (Battaglia et al. 2001)

$$h(k) = \begin{cases} -\omega_k(\alpha)/h^{\alpha}, & k = 1, 2, \ldots, M \\ 0, & k = 0 \end{cases} \tag{4}$$

A positive real part for fractional differentiation order α is chosen for (3), so the fractional integral (1) can be computed. Definition (2) shows that the fractional integral of a function takes into account the past of the function f. More details about the definition of the fractional differentiation are given in Oldham and Spanier (1974) and Prodlubny (2002).

3. Properties of the differentiated image histogram

In this section the statistical properties and the range amplitude variation of the differentiated image histogram are presented.

3.1. Histogram range variation

To illustrate the interpretation of the fractional differentiated function with negative or positive order (also called right-sided fractional integral), a non-negative and causal function f is considered.

The result of applying the digital fractional differentiation (DFD) to the function considered is shown in Fig. 1. Different plots of the fractional differentiated function, obtained with different fractional differentiation orders, are presented in Fig. 1. The application of the fractional differentiation provides a compressed

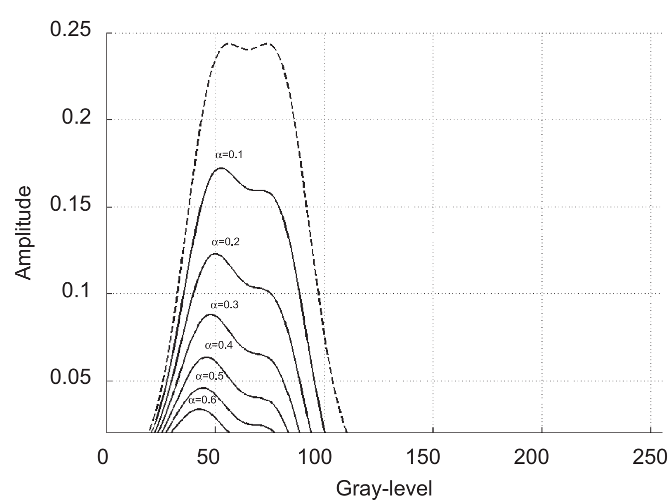

Fig. 1. Decreasing of amplitude range with $\alpha > 0$. Original causal and non-negative function (dashed line) and fractional differentiated function (continuous line) with α varying from 0.1 to 0.6 (step = 0.1).