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Application of practical fuzzy arithmetic to fuzzy internal model control

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ABSTRACT

This paper applies a new fuzzy arithmetic of interval calculus and fuzzy quantities to automatic control. Practical results are obtained which overcome those based on the extension principle or α-cuts. The proposed approach is based on a different representation of fuzzy numbers, though most common arithmetic operators cannot be directly applied for designing a fuzzy controller due to the unjustified overestimation effect. To avoid this phenomenon, a procedure based on an ‘‘exact'' resolution calculus is proposed, whose solutions allow creating a fuzzy internal model control scheme. The validity of the new method is illustrated by a real-time educational engineering application on classical control design: a coupled tanks system.

1. Introduction

Human reasoning frequently relies on knowledge as well as on uncertain data (inaccurate, imprecise, incomplete, inexact, approximate or probabilistic). While automatic control tools make technologically possible to solve many problems based on precise knowledge, handling uncertain information still remains a shady area. This is the case when a human control operator is inside the control loop and/or available information is patchy (missing, etc.), for which standard working tools often show to be inappropriate for data processing. Several theories dealing with uncertainty and imprecision allow these problems to be treated, for example, probability theory, interval theory based on Moore's arithmetic (Moore, 1966), and the fuzzy set theory introduced by Zadeh (1965). These areas are still the subject of a considerable amount of basic and applied research. Possibility theory – an alternative to the probabilistic approach – was first introduced by Zadeh (1978) and further developed by Dubois and Prade (1988). It is an extension of fuzzy set theory that intends to represent both uncertainty and imprecision in the output through the use of possibility and necessity measures. Another well-known approach is the Dempster–Shafer theory, a mathematical framework to deal with evidence based on belief functions and plausible reasoning (Shafer, 1976).

Applications have become increasingly varied, especially in the area of control (Sala et al., 2005; Jaulin et al., 2001; Kulish and Miranker, 1981). Many of them refer to interval theory (Moore, 1966, 1979; Kulish and Miranker, 1981) while others make use of trapezoidal or triangular fuzzy number-based arithmetic (Dubois and Prade, 1978, 1979; Dubois et al., 2004). The latter are generally based on Zadeh's extension principle (Zadeh, 1965; Dubois and Prade, 1979), fuzzy relations (Sanchez, 1984), or the so-called α-cuts (Zhao and Govind, 1991). However, no general approach allowing common arithmetic operations for fuzzy numbers has been developed.

This paper uses φ-calculus arithmetic (Roger and Lecomte, 1998; Lamara et al., 2006) to solve fuzzy intervals and quantities in automatic control. This arithmetic deals with fuzzy numbers and has more practical interest than the extension principle or the α-cut based methods. Instead of modeling a fuzzy quantity using a "classical" membership function (trapezoidal, triangular, L–R function, Gaussian, etc.), which is very often defined in a subjective way, they are modeled by a cumulative distribution function based on classical statistics, hereby allowing imprecise data and uncertainties to be treated (Roger and Lecomte, 1998; Lamara et al., 2006, 2007; Lamara, 2007). Though existing methods found in the literature are not disregarded, this approach presents a way of reasoning and solving problems which entitles the user to a simpler implementation. Since the proposed method is close to probability and classical statistics, it has led – at the first stage – to the analysis of imprecise data as well as to the extension of statistical tools (Lamara et al., 2007; Lamara, 2007). A complete and coherent arithmetic based on fuzzy logic techniques and interval analysis has been then proposed (Lamara et al., 2006, 2007). For this algebra, a Matlab toolbox was developed which allows the use of operators and functions such as square, square root, etc.

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Nevertheless, as it is usually the case for interval and fuzzy arithmetic, results of cumulative algebra operations can lead to strong overestimations, which, if unjustified, are corrected via a "non pessimistic" calculus (Lamara, 2007). Based on the latter as well as on the inverse of the fuzzy process model, a controller is constructed. Notice that inversion-based approaches only apply to stable systems with a minimum phase behavior (systems whose inverted dynamics are stable). This assumption is therefore made onwards and can be found in several papers concerned with fuzzy internal model control (Hunt and Sbarbaro, 1991; Edgar and Postlethwaite, 2000b; Awais, 2005; Boukezzoula et al., 2003; Vermeiren et al., 2008).

The paper is structured in the following way. In Section 2, φ-calculus arithmetic, i.e. modeling of fuzzy numbers, their realization and extension and the weighted fuzzy fusion operator are recalled. In Section 3, a fuzzy internal control model is proposed based on the inversion of the fuzzy process model $\tilde{M}$. To avoid overestimated results due to fuzzy calculation, an "exact" calculus algorithm is proposed. Simulation results are given to illustrate the feasibility of the proposed method. Finally, a real-time control on the coupled tanks system is presented in Section 4.

2. φ-Calculus arithmetic

This section summarizes φ-calculus arithmetic (Roger and Lecomte, 1998; Lamara et al., 2006), and presents the Weighted Fuzzy Fusion (WFF) operator (Lamara et al., 2007; Lamara, 2007).

2.1. Original modeling of fuzzy numbers

In general, fuzzy numbers are defined as fuzzy quantities (Dubois and Prade, 1988) while in practice, fuzzy intervals and fuzzy numbers are mixed up. Traditionally, a fuzzy number $\tilde{a}$ is modeled by its membership function $\mu_{\tilde{a}}$, which is a non-zero function on a bounded set called *support* and denoted as $Supp(\tilde{a}) \subset \mathbb{R}$. For instance, a triangular fuzzy number (TFN) $\tilde{a}$, is represented by the shorthand triplet $\tilde{a} = (b,\ m,\ c)$ with $\mu_{\tilde{a}}(m) = 1$, where m is called mode and the support is defined by the interval $Supp(\tilde{a}) = [b,\ c]$. Throughout the paper, instead of their membership function, fuzzy numbers will be represented by their distribution function $\varphi_{\tilde{a}}$ which is defined as follows:

$$\varphi_{\tilde{a}}(x) = \frac{\int_{-\infty}^{x} \mu_{\tilde{a}}(t)\,dt}{\int_{-\infty}^{+\infty} \mu_{\tilde{a}}(t)\,dt} \tag{1}$$

Due to this definition, for every fuzzy number $\tilde{a}$, $\varphi_{\tilde{a}}(x) : Supp(\tilde{a}) \to [0,1]$ is a non-decreasing function. The existence of its inverse $\varphi_{\tilde{a}}^{-1} : [0,1] \to Supp(\tilde{a}) = [\underline{a}, \overline{a}]$ is therefore guaranteed and will play an important role in defining the subsequent operations.

Convergence of $\int_{-\infty}^{+\infty} \mu_{\tilde{a}}(t)\,dt$ (finite cardinality of $\tilde{a}$) follows from considering only membership functions on a compact support $I = [\underline{a},\ \overline{a}] \subset \mathbb{R}$. The set of fuzzy numbers thus represented is called Φ; it is isomorphic to the set of increasing monotonous functions $m_{\tilde{a}}(x) : Supp(\tilde{a}) \to [0,\ 1]$. Definition of fuzzy number model $\tilde{a} \in \Phi$ is therefore restricted to

$$\varphi_{\tilde{a}} = \begin{cases} 0 & \text{if } x \le \underline{a} \\ \varphi_{\tilde{a}}(x) & \text{if } x \in [\underline{a},\ \overline{a}] \\ 1 & \text{if } x \ge \overline{a} \end{cases} \tag{2}$$

Five membership functions (singleton, interval, triangular, sigmoid and trapezoidal fuzzy numbers) are shown in the first row of Figs. 1 and 2, while their corresponding distribution functions appear in the second row. Notice that the distribution of a singleton is a step function while that of an interval is a ramp between its bounds.

2.2. Fuzzy realization and extension

In a very concrete way – especially for control – it is often necessary to define relations between crisp numbers and fuzzy numbers, i.e. from Φ to $\mathbb{R}$ and $\mathbb{R}$ to Φ. The first one, called *realization*, allows defining which crisp number is associated with a given fuzzy number. The second one, called *extension*, allows defining a fuzzy number from a crisp value. In fuzzy control these concepts correspond to defuzzification and fuzzification, respectively (Dubois and Prade, 2005).

Of course, these operations are not unique. For example possible realizations of a fuzzy number $\tilde{a}$ are as follows:

- $R_{med}(\tilde{a})$: median realization. For a distribution function $\varphi_{\tilde{a}}$ it corresponds to the real number a_0 verifying $\varphi_{\tilde{a}}(a_0) = 0.5$, i.e.

$$R_{med}(\tilde{a}) = \varphi_{\tilde{a}}^{-1}(0.5) \tag{3}$$

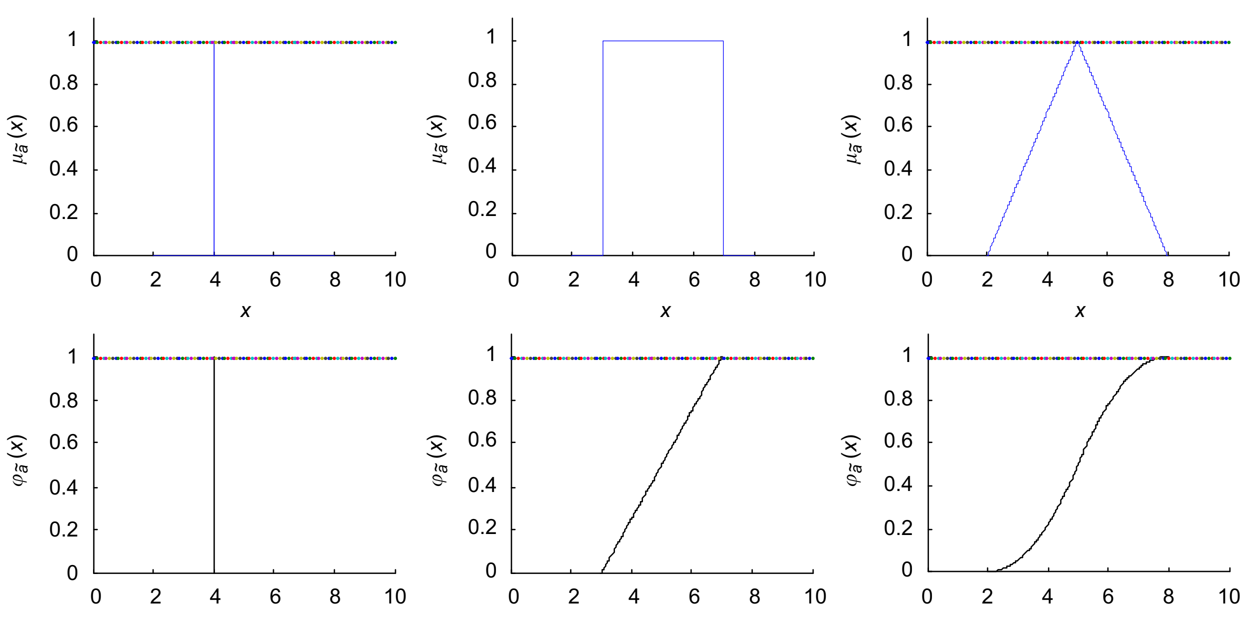

Fig. 1. Modeling of singleton, interval and triangular fuzzy numbers.