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# Observer-based robust adaptive interval type-2 fuzzy tracking control of multivariable nonlinear systems

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ABSTRACT

In this paper, in order to deal with training data corrupted by noise or rule uncertainties, a new observer-based indirect adaptive interval type-2 fuzzy controller is developed for nonlinear MIMO systems involving external disturbances using fuzzy descriptions to model the plant. Based on the universal approximation theorem, a fuzzy logic controller equipped with a training algorithm is proposed such that the tracking error, because of the matching error and external disturbance, is attenuated to an arbitrary desired level using the $H^{\infty}$ tracking design technique. Simulation results show that the interval type-2 fuzzy logic system can handle unpredicted internal disturbances—data uncertainties, very well, but the adaptive type-1 fuzzy controller must expend more control effort in order to handle noisy training data. In the meantime, the adaptive fuzzy controller can perform successful control and guarantee that the global stability of the resulting closed-loop system and the tracking performance can be achieved.



## 1. Introduction

During the past decades, fuzzy sets and their associated fuzzy logic have supplanted conventional technologies in many scientific applications and engineering systems, especially in control systems and pattern recognition. We have also witnessed a rapid growth in the use of fuzzy logic in a wide variety of consumer products and industrial systems. Since 1985, there has been a strong growth in their use for dealing with the control of, especially nonlinear, time varying systems. For instance, fuzzy controllers have generated a great deal of excitement in various scientific and engineering areas, because they allow for ill-defined and complex systems rather than requiring exact mathematical models (Castro, 1995; Chen et al., 1996; Rovithakis and Christodoulou, 1994; Narendra and Parthasarathy, 1990). The most important issue for fuzzy control systems is to deal with the guarantee of stability and control performance, and recently there have been significant research efforts on the issue of stability in fuzzy control systems (Wang, 1993, 1994; Spooner and Passino, 1996).

An adaptive controller differs from an ordinary controller in that the controller parameters are variable, and there is a mechanism for adjusting these parameters online based on signals in the system. Some adaptive control design techniques for feedback linearizable nonlinear systems have already been proposed (Sastry and Isidori, 1989; Marino and Tomei, 1993a, 1993b; Slotine and Li, 1991). The central idea of the feedback linearization is to transform algebraically a nonlinear system dynamics into a (fully or partial) linear one, so that linear control methodologies can be applied. Adaptive controllers possess the essential ability to cope with unavoidable challenges imposed by internal uncertainties, as well as by external environmental uncertainties. Therefore, it is an important subject to design a robust adaptive controller to deal with a nonlinear system with uncertainties. In order to deal with increasingly complex systems, to accomplish increasingly demanding design requirements and the need to attain these requirements with less precise advanced knowledge of the system and its environment, many researchers were compelled to look for more applicable methods.

Recently, based on the feedback linearization technique (Ma and Sun, 2000), adaptive fuzzy control schemes have been introduced to deal with nonlinear systems. Based on the universal approximation theorem, a globally stable adaptive fuzzy controller is synthesized via the combination of IF-THEN rules. The Lyapunov synthesis approach is used to tune free parameters of the adaptive fuzzy controller by output feedback control law and adaptive law. The adaptive fuzzy controllers are classified into two categories: direct and indirect adaptive fuzzy controllers (Park and Yu, 1994). Furthermore, direct adaptive fuzzy controllers use fuzzy logic systems (FLSs) as controllers (Wang et al., 2002a, 2002b); therefore, linguistic fuzzy control rules can be directly incorporated into the controllers. On the other hand,

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indirect adaptive fuzzy controllers use fuzzy logic systems to model the plant and construct the controllers assuming that the fuzzy logic systems represent the true plant (Leu et al., 1999a, 1999b; Wang et al., 1995, 2002a, 2002b); therefore, fuzzy IF-THEN rules describing the plant can be directly incorporated into the indirect adaptive fuzzy controller.

Quite often, the information that is used to construct the rules in an FLS is uncertain. There are three possible ways of rule uncertainty (Karnik et al., 1999; Liang and Mendel, 2000; Mendel and John (2000), Mendel et al., 2006): (i) the words that are used in antecedents and consequents of rules can mean different things to different people; (ii) consequents obtained by polling a group of experts will often be different for the same rule because the experts will not necessarily be in agreement; and (iii) noisy training data. Therefore, antecedent or consequent uncertainties translate into uncertain antecedent or consequent membership functions. Type-1 FLSs are unable to handle rule uncertainties directly, since their membership functions are type-1 fuzzy sets. On the other hand, type-2 FLSs involved in this paper whose antecedent or consequent membership functions are type-2 fuzzy sets can handle rule uncertainties. A type-2 FLS is characterized by IF-THEN rules, but its antecedent or consequent sets are type-2. Hence, type-2 FLSs can be used when the circumstances are too uncertain to determine exact membership grades such as when training data are corrupted by noise.

The type-2 FLS has been successfully applied to fuzzy neural networks (Wang et al., 2004), VLSI testing (Lin, 2009c), and fuzzy controller designs (Wu and Tan, 2006; Wang et al., 2007; Castillo et al., 2006a, 2006b; Lin et al., 2005; Hsiao et al., 2008; Ho et al., 2008; Ross, 2004). An indirect adaptive interval type-2 fuzzy control is proposed in (Kheireddine et al. (2007). Moreover, direct and indirect adaptive interval type-2 fuzzy control is developed in (Lin et al., 2009a, 2009b) for a multi-input/multi-output (MIMO) nonlinear system. In Kheireddine et al. (2007) and Lin et al. (2005), the full state must be assumed to be available for measurement; this assumption may not hold in practice either because the state variables are not accessible for direct connection or because sensing devices or transducers are not available. In this paper, our main objective is to create a technique for designing a state observer-based (Park and Yu, 1994) indirect adaptive interval type-2 fuzzy controller for a model-free nonlinear MIMO system with external disturbances and noisy training data such that the $H^\infty$ tracking performance can be achieved. The illustration example shows that the interval type-2 fuzzy logic system can handle unpredicted internal disturbance—data uncertainties, very well. Moreover, the overall adaptive control scheme not only guarantees global stability, but also the tracking error due to the matching error and external disturbance is attenuated to an arbitrary desired level by using the $H^\infty$ tracking design technique.

This paper is organized as follows. First, the problem formulation is presented in Section 2. A brief description of the interval type-2 fuzzy logic system is then introduced in Section 3. In Section 4, the observer-based indirect adaptive interval type-2 fuzzy controller design for MIMO systems is given and Lyapunov stability theorem is adopted to testify the stability of the type-2 controller system. A simulation example to demonstrate the performance of the proposed method is provided in Section 5. Section 6 gives the conclusions of the advocated design methodology.

## 2. Problem formulation

Consider a class of nonlinear multi-input/multi-output (MIMO) dynamic systems described by the following differential equations (Tong et al., 2000, 2005):

$$x_i^{(n_i)} = f_i(\underline{x}) + \sum_{j=1}^{p} g_{ij}(\underline{x}) u_j + d_i, \quad i = 1,2,\ldots,p$$
$$y_i = x_i \tag{1}$$

where $u \triangleq [u_1, u_2, \ldots, u_p]^T$ and $y \triangleq [y_1, y_2, \ldots, y_p]^T$ are the control input and output of the system, respectively, $f_i(\underline{x})$ and $g_{ij}(\underline{x})$ for $i, j$=1, 2, …, $p$ are unknown smooth but bounded nonlinearities, $n_i$ is a positive integer, $d \triangleq [d_1, d_2, \ldots, d_p]^T$ is the external disturbance vector, and $x \triangleq [y_1, y_1', \ldots, y_1^{(n_1-1)}, \ldots, y_p, y_p' \ldots, y_p^{(n_p-1)}]^T$. The control objective is to force the system output $y$ to follow the given reference signal $y_r \triangleq [y_{r1}, y_{r2}, \ldots, y_{rp}]^T$ under plant uncertainties and external disturbances. Let $G(\underline{x}) \triangleq [G_1(\underline{x}), G_2(\underline{x}), \ldots, G_p(\underline{x})]$ and $G_i(\underline{x}) \triangleq [g_{1i}(\underline{x}), g_{2i}(\underline{x}), \ldots, g_{pi}(\underline{x})]^T$. Hence, (1) can be rewritten as

$$\begin{bmatrix} y_1^{(n_1)} \\ y_2^{(n_2)} \\ \vdots \\ y_p^{(n_p)} \end{bmatrix} = \begin{bmatrix} f_1(\underline{x}) \\ f_2(\underline{x}) \\ \vdots \\ f_p(\underline{x}) \end{bmatrix} + G(\underline{x}) \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_p \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_p \end{bmatrix} \tag{2}$$

If $f_i(x)$ and $g_{ij}(x)$ are known and free of external disturbance, i.e. $d_i$=0, since det $G(\underline{x}) \neq 0$ and based on the equivalent approach, the control law can be obtained as

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_p \end{bmatrix} = G^{-1}(\underline{x}) \left( - \begin{bmatrix} f_1(\underline{x}) \\ f_2(\underline{x}) \\ \vdots \\ f_p(\underline{x}) \end{bmatrix} - \begin{bmatrix} \underline{k}_{1c}^T \underline{e}_1 \\ \underline{k}_{2c}^T \underline{e}_2 \\ \vdots \\ \underline{k}_{pc}^T \underline{e}_p \end{bmatrix} + \begin{bmatrix} y_{r1}^{(n_1)} \\ y_{r2}^{(n_2)} \\ \vdots \\ y_{rp}^{(n_p)} \end{bmatrix} \right) \tag{3}$$

where $e_1 = y_1 - y_{r_1}, e_2 = y_2 - y_{r_2}, \ldots, e_p = y_p - y_{r_p}$, $\underline{e}_i \triangleq [e_i, \dot{e}_i, \ldots, e_i^{(n_i-1)}]^T$ and feedback gain vector $\underline{k}_{ic}^T = [k_{i1}, k_{i2}, \ldots, k_{in_i}]$. Inserting (3) into (2) and after simple manipulation, we have

$$\begin{bmatrix} e_1^{(n_1)} + k_{1n_1} e_1^{(n_1-1)} + \cdots + k_{11} e_1 \\ e_2^{(n_2)} + k_{2n_2} e_2^{(n_2-1)} + \cdots + k_{21} e_2 \\ \vdots \\ e_p^{(n_p)} + k_{pn_p} e_p^{(n_p-1)} + \cdots + k_{p1} e_p \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4}$$

If all the coefficients $k_{ij}$ are chosen such that all polynomials in (4) are Hurwitz, which implies that $\lim_{t\to\infty} e_i(t) = 0$, the main control objective is achieved. However, $f_i(\underline{x})$ and $g_{ij}(\underline{x})$ are unknown, the ideal controller (3) cannot be implemented, and not all system states $\underline{x}$ can be measured. We have to design an observer to estimate the state vector $\underline{x}$.

### 2.1. State observer scheme

Replacing the functions $f_i(\underline{x})$, $g_{ij}(\underline{x})$ and error vector $\underline{e}_i$ in (3) by estimation functions $f_i(\hat{\underline{x}}), g_{ij}(\hat{\underline{x}})$, and error vector $\hat{\underline{e}}_i$, the control law (3) is rewritten as

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_p \end{bmatrix} = G^{-1}(\hat{\underline{x}}) \left( - \begin{bmatrix} f_1(\hat{\underline{x}}) \\ f_2(\hat{\underline{x}}) \\ \vdots \\ f_p(\hat{\underline{x}}) \end{bmatrix} - \begin{bmatrix} \underline{k}_{1c}^T \hat{\underline{e}}_1 \\ \underline{k}_{2c}^T \hat{\underline{e}}_2 \\ \vdots \\ \underline{k}_{pc}^T \hat{\underline{e}}_p \end{bmatrix} + \begin{bmatrix} y_{r1}^{(n_1)} \\ y_{r2}^{(n_2)} \\ \vdots \\ y_{rp}^{(n_p)} \end{bmatrix} \right) \tag{5}$$

**Assumption 1.** The system $G(\hat{\underline{x}})$ as defined above is nonsingular, i.e. $G^{-1}(\hat{\underline{x}})$ exists and is bounded for all $\hat{\underline{x}} \in U_{\hat{\underline{x}}}$, where $U_{\hat{\underline{x}}} \subset R^n$ and $n_1 + n_2 + \cdots + n_p = n$. Also, external disturbance is bounded, i.e. $|d_i| \le d_{im}$, where $d_{im}$ is the upper bound of noise $d_i$.