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Multiscale fuzzy Kalman filtering

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Abstract

Measured data are usually contaminated with errors which sometimes mask their important features. Therefore, data filtering is needed for effective utilization of such measurements. For nonlinear systems which can be described by a Takagi–Sugeno (TS) fuzzy model, several fuzzy Kalman (FK) filtering algorithms have been developed to extend Kalman filtering to such systems. Also, multiscale representation of data is a powerful data analysis tool, which has been successfully used to solve several data filtering problems. In this paper, a multiscale fuzzy Kalman (MSFK) filtering algorithm, in which multiscale representation is utilized to improve the performance of fuzzy Kalman filtering, is developed. The idea is to apply FK filtering at multiple scales to combine the advantages of the FK filter with those of the low pass filters used in multiscale data representation. Starting with a fuzzy model in the time domain, a similar fuzzy model is derived at each scale using the scaled signal approximation of the data obtained by stationary wavelet transform (SWT). These multiscale fuzzy models are then used in FK filtering, and the FK filter with the least cross validation mean square error among all scales is selected as the optimum filter. Also, theoretically, it has been shown that applying FK filtering at a coarser scale than the time domain is equivalent to using a time-averaged FK filter. Finally, the performance of the developed MSFK filtering algorithm is illustrated through a simulated example.

Keywords: Kalman filtering; Fuzzy systems; Multiscale representation

1. Introduction

Fuzzy systems have been widely used and are known to perform well in modeling nonlinear dynamical systems because of their accuracy and ability to incorporate predefined knowledge into their estimation (Passino and Yurkovich, 1998; Johansen et al., 2000; Ying, 1998a, b, c; Zeng et al., 2000). A fuzzy system is an approximator which consists of a set of IF–THEN type rules, each of which has a premise and a consequent part. In standard fuzzy systems, the consequent part is a scalar. A more general class of fuzzy systems includes the functional fuzzy systems, which are usually referred to as the Takagi–Sugeno (TS) fuzzy systems (Passino and Yurkovich, 1998; Takagi and Sugeno, 1985). In such systems, the consequent part is a crisp function, which gives the approximator the ability to incorporate knowledge about the model.

Fuzzy models have been found very useful for control purposes due to their ability to describe complex systems in an efficient manner. However, in order to achieve good fuzzy control, reliable state estimation is essential. The problem of state estimation in systems which can be described by fuzzy models has received little attention from researchers. For example, the authors in Feng et al. (1997), Ma et al. (1998) and Tanaka et al. (1998) have addressed the noise-free case of the fuzzy state estimation problem, in which the system is assumed to be unaffected by noise. Also, they require a solution of a set of Riccati equations, which sometimes are very challenging to solve. Other researchers dealt with the noise corrupted systems. For example, the author in Simon (2003) developed a TS fuzzy Kalman (FK) filtering algorithm, in which local Kalman filters are designed for all local sub-models, which are then interpolated to give the overall state estimate. Also, the authors in Chen et al. (1998) developed a fuzzy version of the interval Kalman filter. Their developed algorithm, which they refer to as the FK filter, provides scalar

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estimates of the states and possesses the same recursive mechanism as the standard Kalman filter.

Unfortunately, the above fuzzy filtering techniques are single scale methods because they all assume that the measured process data only contain features with fixed contributions over time and frequency. In practice, however, measured data usually contain multiscale features. For example, a sudden change in the data spans a wide range in the frequency domain and a narrow range in the time domain, while a slow change spans a wide range in the time domain and narrow range in the frequency domain. Filtering such data using single scale methods is not very effective because they do not account for the multiscale nature of the data. Thus, effective filtering of multiscale data requires a multiscale filtering method.

Multiscale wavelet-based thresholding has shown great success when used in batch univariate filtering (Coifman and Donoho, 1995), which was later extended for online processes (Nounou and Bakshi, 1999). Also, multiscale representation has been shown to improve the performances of other model based filtering methods. For example, the authors in Bakshi et al. (2001) developed a multiscale Bayesian approach for the rectification of linear steady state and dynamic data. The approach applies Bayesian rectification on the wavelet coefficients obtained from the multiscale representation of the data at multiple scales. Also, the authors in Ungarala and Bakshi (2000) developed a multiscale Bayesian error-in-variables approach for the reconciliation of linear dynamic data. Their approach is recursive in both time and scale, and solves an optimization problem using the scaling and wavelet coefficients at every node in the multiscale decomposition tree. The model used in this approach was proposed in Chou et al. (1994), and was later used in estimation (Stephanopoulos et al., 1998), and in model predictive control (Krishnan and Hoo, 1999; Stephanopoulos et al., 2000). However, this scale recursive model was derived using the Haar wavelet and scaling function filters, and therefore, its use is restricted to only multiscale representation using Haar. The reason behind the advantage of the above multiscale methods over the single scale ones is the nice localization properties of the wavelet and scaling basis functions, which allow efficient separation of noise from important features in the data.

The objective of this paper is to develop a multiscale fuzzy Kalman (MSFK) filtering algorithm that combines the advantages of multiscale filtering with those of the FK filter to further improve its performance. The MSFK filtering algorithm relies on applying FK filtering at multiple scales using the scaling function coefficients of the data obtained using stationary wavelet transform (SWT), and then selecting the optimum FK filter, among all scales, which minimizes a cross validation estimation error criterion. The multiscale model used in this MSFK filtering approach is also derived using stationary wavelet transform, which is not restricted to any filter type.

The rest of the paper is organized as follows. In Section 2, the problem of multiscale FK filtering is specifically stated for the TS fuzzy model. Then, in Section 3, a brief description of the FK filter is presented, followed by an introduction to multiscale representation of data and a derivation of the multiscale fuzzy state space model in Section 4. Then, in Section 5, a derivation of the MSFK filter equations is presented, followed by an outline of the MSFK filtering algorithm and an interpretation of this algorithm. Then, the performance of the MSFK filter is demonstrated and compared to that of the standard FK filter through a simulated example in Section 6, followed by few concluding remarks in Section 7.

2. Problem statement

In this paper, the problem of state estimation is addressed from a multiscale perspective for nonlinear dynamic systems which can be described by the following TS fuzzy state space model, whose ith rule is defined as

if $\bar{z}_1(k)$ is $\bar{\alpha}_{1,r1}$ and $\bar{z}_2(k)$ is $\bar{\alpha}_{2,r2}\ldots$ and $\bar{z}_p(k)$ is $\bar{\alpha}_{p,r}$

Then $x_i(k+1) = A_i x(k) + B_i u(k) + \Phi_i w(k)$

$$y_i(k) = C_i x(k) + v(k), \quad \forall i = 1, \ldots, M, \tag{1}$$

where, M is the number of rules. Here, $x(k) \in \Re^n$, $u(k) \in \Re^m$, $y(k) \in \Re^p$, $v(k) \sim N(0,R) \in \Re^p$, and $w(k) \sim N(0,Q) \in \Re^n$ are the actual state, process input, measured output, measurement noise, and process noise, respectively. The premise of each rule is defined such that $\bar{\alpha}_{p,r}$ is the rth linguistic value of the linguistic variable, $\bar{z}_p(k)$, defined over the universe of discourse, U_p. The input to the fuzzy system, $\bar{z}(k) = [\bar{z}_1, \bar{z}_2, \ldots, \bar{z}_p]^{\mathrm{T}}$, is a p-dimensional vector that can be a function of past states, inputs, outputs, or any external input. On the other hand, the consequent part of each rule is a crisp function that is defined as a state space discrete time-invariant system. Assume that $\xi_i(\bar{z}(k))$ represents the certainty that the premise of the ith rule matches the input information. For simplicity, $\xi_i(k)$ is used instead of $\xi_i(\bar{z}(k))$. Assuming that $0 \leqslant \xi_i(k) \leqslant 1$, $\forall i = 1, 2, \ldots, M$, and $\sum_{i=1}^{M} \xi_i(k) \neq 0$, then $\mu_i(k)$, which is the certainty of the ith rule, can be defined as

$$\mu_i(k) = \frac{\xi_i(k)}{\sum_{i=1}^{M} \xi_i(k)},$$

where $\sum_{i=1}^{M} \mu_i(k) = 1$. With this in mind, the overall fuzzy model can be thought of as nonlinear interpolator between M linear systems such that $x(k) = \sum_{i=1}^{M} \mu_i(k) x_i(k)$ and $y(k) = \sum_{i=1}^{M} \mu_i(k) y_i(k)$. Hence, the Takagi–Sugeno fuzzy system can be expressed as

$$x(k+1) = \sum_{i=1}^{M} \mu_i(k)[A_i x(k) + B_i u(k) + \Phi_i w(k)],$$

$$y(k) = \sum_{i=1}^{M} \mu_i(k)[C_i x(k) + v(k)], \tag{2}$$