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Multi-objective genetic algorithms: A way to improve the convergence rate

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Abstract

Multi-objective optimization is generally a time consuming step of the design process. In this paper, a Pareto based multi-objective genetic algorithm is proposed, which enables a faster convergence without degrading the estimated set of solutions. Indeed, the population diversity is correctly conserved during the optimization process; moreover, the solutions belonging to the frontier are equally distributed along the frontier. This improvement is due to an extension function based on a natural phenomenon, which is similar to a cyclical **e**pidemic which happens every **N** generations (eN-MOGA). The use of this function enables a faster convergence of the algorithm by reducing the necessary number of generations.

Keywords: Multi-objective problems; Multi-objective genetic algorithms (MOGA); Pareto frontier; Elitism

1. Introduction

Nowadays there is a trend, notably in the industry, towards complex products which span over several engineering problems and disciplines. Actually, besides the traditional economic point of view, more recent industrial requirements, such as robustness and performance of the design, have become an important characteristic of the optimization process. In this context, real-world engineering design problems involve simultaneous optimizations of several objectives. At the beginning of the design process the set of objectives is unclear and the designer has to define them as precisely as possible. Often, these objectives are conflicting.

Since the 1980s computational computer capabilities have increased. Modelization softwares have benefited from that fact and are now able to perform complex and precise simulations. Conversely, they have simultaneously become more and more time consuming. As an example, one can cite simulations performed in fields like aeronautics, hydrodynamics (fluid simulation) and cars crashing which use finite element models with huge numbers of elements. Because the product "time to market" tends to be minimized, computation time of optimization techniques, like any other time consuming steps of the design process, has become an important economic criterion in the whole project.

In this context, the duration of the optimization step has to stay as reduced as possible in order not to become appalling with regard to the total time of the project in which it takes part. This fact appears more important insofar as the preparation time of the optimization problem is often long. At the present time, several industrial domains such as "real time control" which require extremely fast optimization techniques cannot use optimization techniques based on genetic algorithms (GAs) because of this computing time.

In order to shorten this time—which can be approached as the product of the number of evaluations by the function evaluation time—we have two solutions. The first one consists in reducing the function evaluation time: one way to reach this goal is the use of response surfaces, which gives an approximation of the function. The second

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possibility aims at reducing the number of evaluations required to reach the optimized solution.

As opposed to single-objective optimization problems which accept one single optimum solution, multi-objective optimization problems propose a set of alternative optimum solutions named the Pareto frontier. In order to solve multi-objective problems, the designer can choose between two ideologically opposed sets of techniques. The first set contains single-objective optimization techniques which consist in building a function which aggregates all the objectives of the problem in a single function which is supposed to represent designer's preferences information. Such techniques provide a unique solution which is strongly linked with the aggregation function. Consequently, single-objective techniques can be considered as "a priori" methodologies. The second set contains multi-objective optimization techniques which provide a set of alternative solutions. They are based on an "a posteriori" articulation of preference information in order to make a choice within a set of optimum solutions. In many cases, multiple objective problems are aggregated into one single overall objective function (OF) (Fig. 1). Some optimization techniques available in the operational research field enable the designer to consider each objective separately. Two kinds of optimization methods exist: derivative and non-derivative. The second ones are more suitable for general engineering design problems that (a) do not need any derivatives of the OF and (b) are more likely to explore the whole design space. Since the end of the 1980s, GAs, a non-gradient based method, have grown in popularity (Goldberg, 1989; Colette et al., 2000). This method can also take into account problems with constraints (Sarker et al., 2001). As they seem robust enough to identify multiple optimal solutions and handle multi-modal functions we have decided to use them as an optimization method. Therefore, this paper focusses on multi-objective genetic algorithms (MOGA) and more specifically on a methodology to reduce the number of evaluations of OF and therefore improve the convergence speed.

The present work is structured into four sections. The first section is concerned with multi-objective problems and the Pareto representation of their solutions. The second introduces GA methodologies. The third one describes the improvement we propose in order to enable a faster convergence of the algorithm and an academic example based on the well-known Deb's functions (Deb, 1999). Finally, the last section presents an application on a classical engineering problem: the design of a plane frame structure.

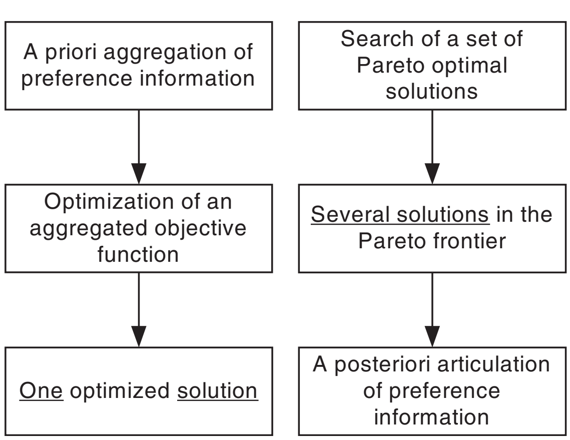

Fig. 1. Single-objective optimization methods vs multi-objective.

2. Multi-objective problems: definition and formulation

2.1. General definition of a multi-objective problem

A multi-objective optimization can be defined as the problem of finding a set of design variables (DV) which optimizes a set of OF and simultaneously satisfies a set of constraint functions. A multi-objective optimization problem can be expressed as follows:

Find DV set:

$$\mathbf{x} = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in (\mathrm{DVS}), \tag{1}$$

which minimizes OF:

$$\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x}))^{\mathrm{T}} \in (\mathrm{OFS}) \tag{2}$$

and simultaneously satisfies constraints:

$$\begin{cases} h_i(\mathbf{x}) = 0, & i \in [1, \ldots, q_h] \text{ (equality constraints)}, \\ g_i(\mathbf{x}) \leqslant 0, & i \in [1, \ldots, q_g] \text{ (inequality constraints)}, \end{cases} \tag{3}$$

where:

- n is the number of DV (optimization parameters) which belong to the design variables space (DVS);
- k is the number of OF to be optimized; OF are included in the objective function space (OFS). The space of DV can contain both discrete and continuous variables;
- q_h is the number of equality constraint functions;
- q_g is the number of inequality constraint functions.

2.2. Pareto frontier concept

If we note $f_1^*, f_2^*, \ldots, f_k^*$ as the individual minima of each respective OF, the utopian solution, $\mathbf{f}^* = (f_1^*, f_2^*, \ldots, f_k^*)$, is the best theoretical solution which simultaneously minimizes all the objectives. Nevertheless, this utopian solution is rarely feasible because of the existence of constraints. Often $\mathbf{f}^*$ does not belong to the OFS and we use the Pareto frontier to define a set of solutions instead of the optimum solution (Fig. 2). The Pareto-optimality is defined as a set, $\mathscr{F}_{\mathbf{P}}$, where every element, $\mathbf{f}_{\mathbf{P,i}}$, is a solution of the problem—defined by Eqs. (1)–(3)—for which no other solutions can be better with regard to all the OF. A solution in a Pareto-optimal set cannot be considered better than the others within the set of solutions without including preference information.

For a minimization problem, considering two solution vectors $\mathbf{x}$ and $\mathbf{y}$, one says that $\mathbf{x}$ is contained in the Pareto