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ε constrained differential evolution with pre-estimated comparison using gradient-based approximation for constrained optimization problems

Wenchao Yi[a], Xinyu Li[a], Liang Gao[a,*], Yinzhi Zhou[a], Jida Huang[b]

[a] State Key Laboratory of Digital Manufacturing Equipment & Technology, Huazhong University of Science and Technology, Wuhan 430074, PR China
[b] Industrial and Systems Engineering Department, University at Buffalo-SUNY, Buffalo, NY 14260, USA

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ABSTRACT

Many real-world problems can be categorized as constrained optimization problems. So, designing effective algorithms for constrained optimization problems become more and more important. In designing algorithms, how to guide the individuals moving more efficiently towards the feasible region is one of the most important aspects on finding the optimum of constrained optimization problems. In this paper, we propose an improved ε constrained differential evolution, which combines with pre-estimated comparison gradient based approximation. The proposed algorithm uses gradient matrix to determine whether the trail vector generated by differential evolution algorithm is worth using the fitness function to evaluate it or not. Pre-estimated comparison gradient based approximation is used as a detector to find the promising offspring and in this way can we guide the individuals moving towards the feasible region. The proposed method is tested both on twenty-four benchmark functions and four well-known engineering optimization problems. Experimental results show that the proposed algorithm is highly competitive in comparing with other state-of-the-art algorithms. The proposed algorithm offers higher accuracy in engineering optimization problems for constrained optimization problems.

1. Introduction

In engineering and scientific fields, a lot of problems can be summarized as the constrained optimization problems (COPs), such as water pumping system (Babu & Angira, 2003), pin-joint plane frame design (Majid, 1974), gear train design (Sandgran, 1990), and parameter estimation in engineering (Jiang, Maskell, & Patra, 2013) etc. In general, the mathematical model of COPs can be given as follows:

$$\text{minimize } f(\vec{x})$$
$$\text{s.t.} \begin{cases} g_j(\vec{x}) \le 0, & j = 1, \dots, q \\ h_j(\vec{x}) = 0, & j = q+1, \dots, m \end{cases},$$

where $\vec{x} = (x_1, x_2, \dots, x_n) \in \Omega \subseteq S$ is n-dimensional decision vector generated in the decision space S. $f(\vec{x})$ denotes the objective function. $g_j(\vec{x})$ is the jth inequality constraint. $h_j(\vec{x})$ is the jth equality constraint. Ω denotes the feasible region. The equality constraints can be transformed into the inequality constraint by adding a tolerance value δ as follows:

$$\left|h_j(\vec{x})\right| - \delta \le 0.$$

The constraint violation value $G_j(\vec{x})$ of $\vec{x}$ is calculated as follows:

$$G_j(\vec{x}) = \begin{cases} \max\left\{0, g_j(\vec{x})\right\}, & 1 \le j \le l \\ \max\left\{0, \left|h_j(\vec{x}) - \delta\right|\right\}, & l+1 \le j \le p \end{cases}.$$

Differential evolution (DE) algorithm, as an effective evolutionary algorithm, has attracted many researchers making improvement since proposed by Storn and Price (1995). Several DE variants had been proposed in dealing with COPs. Mohanmed and Sabry (2012) proposed a modified DE algorithm, in which a novel mutation operator based on the difference vector between the best and the worst individual in the population was proposed. Zhou, Liu, Gao, and Li (2011) introduced a modified DE algorithm based penalty function method by modifying the control parameters in an adaptive way. Wang and Cai (2012b) presented a dynamic hybrid framework based on DE algorithm. Global search and local search models were proposed and these two models were dynamically executed in the evolutionary process.

Except the algorithm part, the constraint handling technique also has a great impact on solving COPs. Due to various types and amount

* Corresponding author. Tel.: +86 27 87557742; fax: +86 27 87543074.
E-mail addresses: janety1989@gmail.com (W. Yi), lixinyu@mail.hust.edu.cn (X. Li), gaoliang@mail.hust.edu.cn (L. Gao), leitezhou@gmail.com (Y. Zhou), jidahuan@buffalo.edu (J. Huang).

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of constraints, COPs are usually very difficult to be solved successfully. The solving methods mainly can be summarized into following three categories:

(1) **Traditional methods.** This kind of approaches transforms the constraints into objective function, such as the penalty function method. Penalty function method is a simple but effective method in solving COPs. Michalewicz (1995) proposed the static penalty method. However, it is difficult to find a proper penalty factor for different constraints.
(2) **Multi-objective concept based technique.** This technique converts the constraints into another objective function. Usually, two objective functions are considered, one is the original objective function, while the other is the constraints violation objective function. In this way, the COPs can be converted into unconstraint optimization problems. For example, Qu and Suganthan (2011) presented a multi-objective DE algorithm with emblem of several constraint handling techniques. Wang and Cai (2012a) proposed a method using multi-objective optimization technique to make the comparisons between individuals.
(3) **Separately handling the constraint and objective function based methods.** This kind of methods offers another way in solving COPs. Several researches have been made in this category. Deb (2000) proposed pair-wise comparison rules to assign preference on feasible individuals. Takahama and Sakai (2005) proposed the α constrained method that the α level comparison is introduced. Since the feasibility is more important than the minimization of the objective function, the α constrained method define a lexicographic order rule that gives priority to constraint violation $\phi(x)$. Based on the α constrained method, Takahama and Sakai (2006) proposed the ε constrained differential evolution (εDE) in which an advanced lexicographic order rule was developed.

For category (1), it is still difficult for penalty function method to find the best suitable penalty factors. Although some dynamical penalty factor methods had been proposed (Coello Coello, 2000; Paszkowicz, 2009; Tasgetiren & Suganthan, 2006), penalty factor is still a problem dependent issue which is difficult to be handled. For category (2), multi-objective optimization problems are difficult to be solved. Hence, converting the constrained optimization problems into multi-objective optimization problems may be more difficulty to be solved.

As a representative of category (3), εDE algorithm is a highly competitive method in solving COPs. When the feasible region is small, using εDE method can make the individuals with small constraint violation to ε-feasible ones. Hence, the method can guide the population moving towards the feasible region efficiently. Takahama has proposed several improved version based on εDE algorithm. The εDE algorithm with an archive and gradient-based mutation (Takahama & Sakai, 2010) was introduced in 2010, where gradient matrix is calculated to help to obtain the new position of each individual. Using this method, the individual can calculate the approximate or best solution using the pseudoinverse and gradient matrix.

Although the above researches had been done in COPs, there still have many rooms to improve their performances. When COPs are encountered with problems with small feasible region, finding the feasible solution may take precedence over the minimization of the objective function. However, it is also very hard to find the feasible sub-optimal solutions with a large number of fitness function evaluations. Based on the above analyses, the main motivation of this paper is to overcome the drawback of evaluating the individual blindly when the fitness function evaluations are limited. DE algorithm is one of the most highly competitive algorithms among the evolutionary algorithms, which can serve as the searching engine of the COPs. As a result, in this paper, we propose the pre-estimate comparison gradient-based approximation ε constrained differential evolution (εDE-PCGA) algorithm to solve the COPs that belongs to promising category (3). To evaluate the individual more efficiently, the pre-estimate comparison gradient-based approximation (PCGA) is proposed to decide whether the individual is worth of calling the objective function to evaluate or not, which serves as the detector to detect the promising unknown region. Twenty-four benchmark test functions collected from the 2006 IEEE Congress on Evolutionary Computation (CEC 2006) special session on constrained real-parameter optimization problem definitions on evaluation criteria are adopted to demonstrate the effectiveness of the proposed algorithm. In addition, four real world engineering optimization problems are selected to evaluate the performance of the proposed algorithm. The experimental results show that the performance of the proposed algorithm is highly competitive compared with other state-of-the-art algorithms.

The remaining part of this paper is organized as follows. Section 2 introduces the DE and εDE algorithm briefly. The proposed εDE-PCGA is presented in Section 3. The experimental results of CEC 2006 benchmark test functions and four real world engineering optimization problems will be presented in Section 4. Section 5 concludes the paper and future work will be given.

2. ε constrained differential evolution algorithm

εDE is a recently proposed efficient constrained optimization algorithm. In this section, we will first introduce DE algorithm briefly. Then ε constrained method and εDE algorithm will be introduced.

2.1. Differential evolution algorithm

DE algorithm, proposed by Storn and Price (1995), is an efficient direct search evolution algorithm. In DE algorithm, initial population is generated within the feasible region. At generation $G = 0$, the n-dimensional initial population can be expressed as follows:

$$x_{i,G} = \left(x_{i,G}^1, x_{i,G}^2 \ldots, x_{i,G}^n\right),\ i = 1, 2, \ldots, NP;\ x_{i,G}^j \in \left[\mathrm{LB}_j, \mathrm{UB}_j\right], \quad (1)$$

where NP denotes the population size. LB_j and UB_j denote the lower bound and upper bound in jth dimension, respectively.

After initialization, the mutation operator is used to generate the mutant vector $v_{i,G} = (v_{i,G}^1, v_{i,G}^2, \ldots v_{i,G}^n)$ for each individual $x_{i,G}$ (also called target vector). The most widely used mutation operator DE/rand/1 is shown as follows:

$$v_{i,G} = x_{r1,G} + F * (x_{r2,G} - x_{r3,G}). \quad (2)$$

where $r1$, $r2$, and $r3$ are three integers that are mutually different from i and randomly generated within the range [1, NP]. F, called scaling factor, is a predefined parameter.

Then crossover operator is utilized to generate the trail vector $u_{i,G} = (u_{i,G}^1, u_{i,G}^2, \ldots u_{i,G}^n)$. A binomial crossover operator is conducted on the target vector $x_{i,G}$ and mutant vector $v_{i,G}$ as follows:

$$u_{i,G}^j = \begin{cases} v_{i,G}^j, & \text{if } \mathrm{rand}_j(0,1) \le \mathrm{CR}, \text{ or } j = j_{\mathrm{rand}} \\ x_{i,G}^j, & \text{otherwise} \end{cases}$$
$$i = 1, 2, \ldots, NP;\ j = 1, 2, \ldots, n. \quad (3)$$

where j_{rand} is random integer that is generated within the range $[1, n]$. rand_j (0,1) is a random number generated between 0 and 1. CR, called control parameter, is a predefined parameter that generated between 0 and 1. By using j_{rand} we can ensure that the trail vector $u_{i,G}$ differs from target vector $x_{i,G}$.

Then a checking process that Wang et al. (2012) used is adopted to ensure the generated trail vector is generated within the feasible