Portfolio optimization using a credibility mean-absolute semi-deviation model

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ABSTRACT

We introduce a cardinality constrained multi-objective optimization problem for generating efficient portfolios within a fuzzy mean-absolute deviation framework. We assume that the return on a given portfolio is modeled by means of LR-type fuzzy variables, whose credibility distributions collect the contemporary relationships among the returns on individual assets. To consider credibility measures of risk and return on a given portfolio enables us to work with its Fuzzy Value-at-Risk. The relationship between credibility expected values for LR-type fuzzy variables and possibilistic moments for LR-fuzzy numbers having the same membership function are analyzed. We apply a heuristic approach to approximate the cardinality constrained efficient frontier of the portfolio selection problem considering the below-mean absolute semi-deviation as a measure of risk. We also explore the impact of adding a Fuzzy Value-at-Risk measure that supports the investor's choices. A computational study of our multi-objective evolutionary approach and the performance of the credibility model are presented with a data set collected from the Spanish stock market.

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1. Introduction

Financial planning has been considered one of the prototype decision making problems. A wide variety of mathematical programming models have been developed to address problems related to financial management and many optimization techniques have been introduced to solve them. Markowitz (1952) gave the first mathematical formulation of the portfolio selection problem, assuming that the return on every asset is a random variable with a given probability distribution, and that the risk of the investment is measured in terms of the variance of the portfolio. The pioneering mean–variance (MV) model by Markowitz assumes that the vector of returns on assets is multivariate-normally distributed and that the investor prefers lower risks, with this approach leading to a quadratic programming problem. Alternatively, Konno and Yamazaki (1991) proposed a linear optimization model for portfolio selection by using the mean absolute deviation (MAD) around the expected return as a measure of risk; while Cai, Teo, Yang, and Zhou (2000) introduced the minimax rule in the portfolio selection and provided a new l_∞ model based on the maximum absolute deviation as a measure of risk. Recently, the mean absolute deviation has also been used as measure of risk

in a cardinality constrained portfolio selection framework, where genetic algorithms provide efficient portfolios (Chang, Yang, & Chang, 2009).

In the classical modeling approach, an optimal portfolio has to satisfy a kind of balance between maximizing the expected return and minimizing the risk of the investment, respecting the investor's declared preferences. The portfolio selection problem then becomes an optimization problem with multiple objectives and/or additional constraints. Alternative statistical measures of portfolio return, like semi-variance, absolute semi-deviation, skewness and value-at-risk (the worst expected loss over a given horizon), can also be found in the multi-objective portfolio selection models and many multi-criteria tools have been used to find efficient portfolios (see, for instance, Ehrgott, Klamroth, & Schwehm (2004), Steuer, Qi, & Hirschberger (2007) and references therein). Recently, some researchers apply multi-objective evolutionary algorithms to suitably handling these more complex portfolio optimization problems (Anagnostopoulos & Mamanis, 2011; Liagkouras & Metaxiotis, 2014). A comprehensive survey of several heuristic approaches for portfolio management can be found in Metaxiotis and Liagkouras (2012).

Solving the portfolio selection problem must provide both the assets and their corresponding proportions that are an optimal investment, in a certain way. Then, in order to determine an optimal portfolio both optimization techniques and uncertainty

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quantification approaches must be considered. An alternative framework for decision making analysis of the portfolio selection is based on fuzzy set theory, which also allows the incorporation of imperfect knowledge of future market behavior into the modeling approach. To address the fuzzy portfolio selection problem, it is necessary to formulate how to treat the imprecise uncertainty of the portfolio returns. Usually the return on risky individual assets has been approximated using fuzzy numbers with possibility distributions (Carlsson, Fullér, & Majlender, 2002; Vercher, Bermúdez, & Segura, 2007), although recently some authors suggest treating these returns by means of credibility distributions or under the assumption that the returns on assets are random fuzzy variables (Hasuike, Katagiri, & Ishii, 2009; Huang, 2008). In addition, Fuzzy Value-at-Risk and fuzzy skewness measures have been considered providing new portfolio selection model proposals when symmetrical returns on assets are not assumed (see, for instance, Li, Qin, & Kar (2010), Wang, Wang, & Watada (2011), Vercher & Bermúdez (2013) and references therein). Some research has also appeared concerning fuzzy multi-criteria decision making for the portfolio selection problem, where different soft computing approaches have been considered in order to determine the corresponding efficient frontier (Bermúdez, Segura, & Vercher, 2012; Gupta, Inuiguchi, Mehlavat, & Mittal, 2013; Jiménez & Bilbao, 2009).

In this paper, we consider a fuzzy modeling framework to determine efficient portfolios by assuming that the uncertainty of return on a given portfolio is quantified using credibility distributions. The fuzzy risk, skewness and return on the portfolio are then defined by means of credibility moments for the LR-type fuzzy variables, which are built using the historical data set of portfolio returns. We present a credibility mean-absolute semi-deviation model (CMASd) for portfolio selection, which also incorporates the skewness value as a constraint jointly with cardinality and diversification conditions that respect the investors' risk profiles. That is, we present a new fuzzy portfolio selection model in the risk-return trade-off using credibility moments and critical values, and we apply a multi-objective evolutionary algorithm for obtaining the cardinality constrained efficient frontier. Finally, we introduce a DSS strategy based on the Fuzzy Value-at-Risk to determine the most efficient portfolio, for a certain bound of the percentage of loss.

The next section introduces basic notation and some results of fuzzy numbers, fuzzy variables and credibility distributions. Section 3 is devoted to describing the relationship (both equivalences and discrepancies) between credibility expected values for LR-type fuzzy variables and possibilistic moments for LR-fuzzy numbers, when they have the same membership function. In Section 4, we propose the credibility mean-absolute semi-deviation portfolio selection model and emphasize some of the novelties concerning the evolutionary algorithm that we apply to find the approximated Pareto frontier. In Section 5, we illustrate our proposal with a data set from the Spanish stock market.

2. Possibility and credibility distributions: basic background

In this section, we collect definitions, notation and several results from Fuzzy Set Theory. A fuzzy number Q is a convex normalized fuzzy set of the real line, whose membership function μ_Q is piecewise continuous. For $y \in \Re, \mu_Q(y)$ is the grade of membership of y in Q, the closer the value of $\mu_Q(y)$ is to 1, the more it belongs to Q. Following Zadeh's proposals (Zadeh, 1978) a fuzzy number Q induces a possibility distribution that is equal to μ_Q. That is, viewing μ_Q as a possibility distribution the degree of possibility of the statement "Q takes the value y" can be represented

by $\mu_Q(y)$. Let us now introduce the definition of LR-type fuzzy number.

Definition 1 (*Dubois and Prade, 1987; Inuiguchi and Tanino, 2002*). A fuzzy number Q is said to be an LR-type fuzzy number if its membership function has the following form:

$$\mu_Q(x) = \begin{cases} L\left(\frac{A-x}{s_A}\right) & \text{if } -\infty < x \leqslant A \\ 1 & \text{if } A \leqslant x \leqslant B \\ R\left(\frac{x-B}{s_B}\right) & \text{if } B \leqslant x < +\infty \end{cases}$$

where A and B satisfy $A \leqslant B$ and show the lower and upper bounds of the core of Q, respectively; i.e., $[A, B] = \{x \mid \mu_Q(x) = 1\}$. s_A and s_B show the left and right spreads of Q, respectively. L and $R : [0, +\infty) \to [0, 1]$ are reference functions which are non-increasing and upper semi-continuous with $\lim_{t \to +\infty} L(t) = \lim_{t \to +\infty} R(t) = 0$.

A fuzzy number Q is said to be a bounded LR-type fuzzy number if the reference functions are such that the support of Q is bounded; i.e. there exist two real numbers a and $b, a < b$, such that $\{y : \mu_Q(y) > 0\} \subset [a, b]$.

Example 1 (*LR-power fuzzy numbers*). When the reference functions belong to the power family with positive parameters α and β, that is $L(t) = \max(0, 1 - t^\alpha)$ and $R(t) = \max(0, 1 - t^\beta), Q$ is an LR-power fuzzy number. The support of Q is then bounded and given by (a, b), being $a = A - s_A$ and $b = B + s_B$, with $[A, B]$ the core of Q and s_A and s_B their left and right spreads, respectively. Throughout the paper these LR-power fuzzy numbers will be denoted by $Q = (a, A, B, b)_{\alpha, \beta}$.

Note that any trapezoidal fuzzy number is a particular case of an LR-power fuzzy number with $\alpha = \beta = 1$; in addition, if $A = B$ then the number is called triangular.

The possibility and necessity of every fuzzy event (for instance, $\{Q \geqslant y\}$ being y a real number) can be evaluated based on the possibility distribution associated to the corresponding membership function. In particular,

$$\text{Pos}\{Q \geqslant y\} = \sup_{t \geqslant y} \mu_Q(t)$$

and

$$\text{Nec}\{Q \geqslant y\} = 1 - \sup_{t < y} \mu_Q(t)$$

As an alternative measure of the uncertainty of a fuzzy event, Liu and Liu have introduced the concept of a credibility measure (Liu & Liu, 2002):

$$\text{Cr}\{Q \geqslant y\} = \frac{1}{2}(\text{Pos}\{Q \geqslant y\} + \text{Nec}\{Q \geqslant y\})$$

The credibility measure is the basis of credibility theory, and it has the property of self-duality, which is:

$$\text{Cr}\{Q \geqslant y\} = 1 - \text{Cr}\{Q < y\}$$

The survey by Liu (2006) is an extensive classical reference on credibility theory (see also the references quoted therein). Let us recall some useful definitions and results on fuzzy variables and credibility distributions.

Definition 2. A fuzzy variable is a function from a credibility space $(\Theta, P(\Theta), \text{Cr})$ to the set of real numbers $\Re$, where Θ is a nonempty set and $P(\Theta)$ is the power set of Θ.

Definition 3. The membership function of a fuzzy variable ξ defined on the credibility space $(\Theta, P(\Theta), \text{Cr})$ is given by: