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Generalized Type-2 Fuzzy Systems for controlling a mobile robot and a performance comparison with Interval Type-2 and Type-1 Fuzzy Systems

Mauricio A. Sanchez [a], Oscar Castillo [b,*], Juan R. Castro [a]

[a] Autonomous University of Baja California, Tijuana, Mexico
[b] Tijuana Institute of Technology, Tijuana, Mexico

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ABSTRACT

The aim of this paper is to show that a Generalized Type-2 Fuzzy Control System can outperform Type-1 and Interval Type-2 Fuzzy Control Systems when external perturbations are present. A Generalized Type-2 Fuzzy System can handle better uncertainty because of the nature of its membership functions, and as such, they are better tailored for situations where external noise is present. To test the noise resilience of Fuzzy Controllers, the design of a Fuzzy Controller for a mobile robot is presented in this paper, in conjunction with three types of external perturbations: band-limited white noise, pulse noise, and uniform random number noise. Noise resilience is measured through different performance indices, such as ITAE, ITSE, IAE, and ISE. Simulation results show that Generalized Type-2 Fuzzy Controllers outperform their Type-1 and Interval Type-2 Fuzzy Controller counterparts in the presence of external perturbations.

1. Introduction

In 1965, when Lofti A. Zadeh first proposed Fuzzy Sets (FSs) (Zadeh, 1965) his vision was set on giving more control over decision making, with his Fuzzy Logic an immeasurable amount of decision making situations could be easily modeled whereas hard logic, using true or false values, could not. This opened a new era in decision making with FSs that have been evolving since its initial days, first starting out with the concept of a Type-1 Fuzzy Logic System (T1FLS), then coming into an Interval Type-2 Fuzzy Logic System (IT2FLS) and finally arriving at the current state of advanced form of FS, which is a Generalized Type-2 Fuzzy Logic System (GT2FLS).

While T1FLSs have been around longer and are not focused on directly modeling uncertainty, just as an IT2FLS and GT2FLS do, they focus on imprecision and give a good representation of knowledge in the form of *IF...THEN* fuzzy rules. This imprecision that T1FSs represent is in the form of membership functions, or linguistic values, and it is a powerful tool which is still widely used, for example, in synthetic aperture radar image change detection (Gong, Su, Jia, & Chen, 2014), conditional density estimation by using probabilistic fuzzy systems (Van den Berg, Kaymak, & Almeida, 2013), predictive control of direct methanol fuel cells (Yang, Feng, & Zhang, 2014), fuzzy clustering via a granular gravitational technique (Sanchez, Castillo, Castro, & Melin, 2014), a support system for sellers in e-marketplaces (Kolomvatsos, Anagnostopoulos, & Hadjiefthymiades, 2014), image segmentation (Othman, Tizhoosh, & Khalvati, 2014), distributed filtering in sensor networks (Su, Wu, & Shi, 2013), indoor localization using WiFi (Garcia-Valverde et al., 2013), etc. Research in T1FLS is still ongoing despite the existence of more advanced FS representations (IT2FS and GT2FS), partly due to the fact that not all decision making situations require such advanced representations and T1FS can do a good enough job; some examples of such current research can be seen in Fuzzy Inference System extensions based on the Lattice theory (Kaburlasos & Kehagias, 2014), adaptive fuzzy interpolation (Yang & Shen, 2011), convex fuzzy partition for deriving order compatible fuzzy relations (Sandri & Martins-Bedê, 2014), fuzzy implications derived from generalized h-generators (Liu, 2013), a linguistic computational model based on discrete fuzzy numbers for computing with words (Massanet, Riera, Torrens, & Herrera-Viedma, 2014), etc.

With the advancement of IT2FLSs, uncertainty could finally be directly incorporated into the Fuzzy Sets. This integrated uncertainty improved the system's resilience against noise and unknown data handling. Although the boom of research with IT2FLSs is

* Corresponding author.
E-mail addresses: mauricio.sanchez@uabc.edu.mx (M.A. Sanchez), ocastillo@tectijuana.mx (O. Castillo), jrcastror@uabc.edu.mx (J.R. Castro).

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recent, there is still much to be explored, some current examples of research are shown by, simplified IT2FLS (Mendel & Liu, 2013), enhanced type-reduction (Yeh, Jeng, & Lee, 2011), fuzzy operations (Hu & Wang, 2014), centroid of triangular en Gaussian IT2FS (Starczewski, 2014), fuzzy model of computing with words (Jiang & Tang, 2014), type-reduction algorithms (Mendel, 2009a), etc. With respect to applications where IT2FLS demonstrate their affinity to uncertainty resilience, there is a multitude of research, for example, vectorization-optimization for noisy data classification (Wu & Huang, 2013), deriving the analytical structure of a broad class of Interval Type-2 Mamdani Fuzzy Controllers (Zhou & Ying, 2013), sliding controller for wing rock systems (Tao, Taur, Chang, & Chang, 2012), spatial analysis (Di Martino & Sessa, 2014), an ELECTRE-based outranking method for multiple criteria group decision making (Chen, 2014), cerebellar model articulation controller for chaos time-series prediction and synchronization (Lee, Chang, & Lin, 2013), fuzzy neural networks (Lin, Liao, Chang, & Lin, 2014), load forecasting (Khosravi & Nahavandi, 2014), etc.

The current state of advancement in Fuzzy Logic is GT2FLSs, these being an improvement over IT2FLSs. In comparison with IT2FS, where the uncertainty is represented as an area, in GT2FS the uncertainty is depicted by a volume, and as such, are more capable of handling uncertainty. As GT2FS research is still fairly new, existing research is fairly limited, some examples of advancements are shown in computing the centroid by means of the centroid-flow algorithm (Mendel, 2011), similarity measures (Hao & Mendel, 2014), hierarchical collapsing method for direct defuzzification (Doostparast Torshizi & Fazel Zarandi, 2014), definition of footprint of uncertainty (Mo, Wang, Zhou, Li, & Xiao, 2014), a fast method for computing the centroid (Wu, Su, & Lee, 2012), enhanced type-reduction (Yeh et al., 2011), monotone centroid flow algorithm for type-reduction (O. Linda and Manic, 2012), conversion from IT2FS to GT2FS (Wagner, Miller, Garibaldi, Anderson, & Havens, 2014), computing with words for discrete GT2FS (Zhao, Li, & Li, 2013), matching GT2FS by comparing the vertical slices (Rizzi, Livi, Tahayori, & Sadeghian, 2013), and formation of GT2FS based on the information granule numerical evidence (Sanchez, Castro, & Castillo, 2013). Available research with applications of GTFSs are even more limited, for example, edge detection for image processing (Melin, Gonzalez, Castro, Mendoza, & Castillo, 2014), fuzzy c-means for uncertain fuzzy clustering (Ondrej Linda and Manic, 2012), face-space approach to emotion recognition (Halder et al., 2013), and multi-criteria group decision making (Naim & Hagras, 2013).

The main contribution of the paper is the proposed approach to achieve Fuzzy Control (FC) by using the novel concepts of GT2FLS in a GT2 Fuzzy Controller (GT2FC) and showing that it outperforms Interval Type-2 Fuzzy Controllers (IT2FC) and Type-1 Fuzzy Controllers (T1FC) in a robotic control application.

This paper is separated into multiple sections, this introduction, then, in Section 2 a description of each type of Fuzzy Sets is shown as well as a revision of the state of the art in Fuzzy Control, Section 3 describes the problem statement of a mobile robot, afterwards in Section 4 there is a sample application with a Fuzzy Controller, which shows the improved resilience to noise as results are obtained with T1FC, then IT2FC, and finally GT2FC. Finally, Section 5 concludes the paper with some remarks about the contribution as well as future work possibilities.

2. Fuzzy systems

Rule-based Fuzzy Logic Systems can be Type-1, Interval Type-2 or Generalized Type-2, depending on the level of uncertainty which wants to be handled. The following sub-sections give a short description of each one.

2.1. Type-1 Fuzzy Inference Systems

The first instance of Fuzzy Logic Systems (FLS) which existed is that of Type-1. Fig. 1 shows a block diagram depicting the main sections of the FLS, which can be perceived as simple and direct. In this case the Fuzzifier takes crisp inputs and maps them into FS; the Inference, based on the Rules, maps Fuzzy Sets from the antecedents to FS from the consequents; finally, the Output Processor defuzzifies and outputs a crisp value.

A Type-1 Fuzzy Sets A, denoted by $\mu_A(x)$ where $x \in X$, is represented by $A = \{(x, \mu_A(x)) \mid x \in X\}$ which is a FS, which takes on values between the interval [0,1]. In this case, Fig. 2 shows a sample T1FS.

The rules for a T1FLS are in the form of Eq. (1), and the relation between the input space and output space is mapped with these rules. Where R^l is a specific rule, x_p is the input p, F_p^l is a membership function on rule l and input p, y is the output on membership function G^l. Both F and G are in the form of $\mu_F(x)$ and $\mu_G(y)$ respectively.

$$R^l: \text{IF } x_1 \text{ is } F_1^l \text{ and } \ldots \text{ and } x_p \text{ is } F_p^l, \text{ THEN } y \text{ is } G^l, \text{ where } l = 1, \ldots, M \quad (1)$$

The inference is first performed rule-wise by Eq. (2) when done with t-norm connectors ($\tilde{*}$). Where μ_{B^l} is the resulting membership function in the consequents per each rule's inference, and Y is the space belonging to the consequents.

$$\mu_{B^l}(y) = \mu_{G^l}(y) \tilde{*} \left\{ \left[\sup_{x_1 \in X_1} \mu_{x_1}(x_1) \tilde{*} \mu_{F_1^l}(x_1) \right] \tilde{*} \ldots \tilde{*} \left[\sup_{x_p \in X_p} \mu_{x_p}(x_p) \tilde{*} \mu_{F_p^l}(x_p) \right] \right\}, y \in Y \quad (2)$$

Finally, the defuzzification process can be performed in multiple ways, all achieving a very similar result, some defuzzifier methods are the centroid, center-of-sums, or heights, described by Eqs. (3)–(5) respectively. Where y_i is a discrete position from Y, $y_i \in Y$, $\mu_B(y)$ is a FS which has been mapped from the inputs, c_{B^l} denotes the centroid on the lth output, a_{B^l} is the area of the set, and $\bar{y}^l$ is the point which has the maximum membership value in the lth output set.

$$y_c(x) = \frac{\sum_{i=1}^{N} y_i \mu_B(y_i)}{\sum_{i=1}^{N} \mu_B(y_i)} \quad (3)$$

$$y_a(x) = \frac{\sum_{l=1}^{M} c_{B^l} a_{B^l}}{\sum_{l=1}^{M} a_{B^l}} \quad (4)$$

$$y_h(x) = \frac{\sum_{l=1}^{M} \bar{y}^l \mu_{B^l}(\bar{y}^l)}{\sum_{l=1}^{M} \mu_{B^l}(\bar{y}^l)} \quad (5)$$

2.2. Interval Type-2 Fuzzy Inference Systems

This type of IT2FLS handles uncertainty directly into its system, whereas a T1FLS cannot. Although it follows the same logic as a

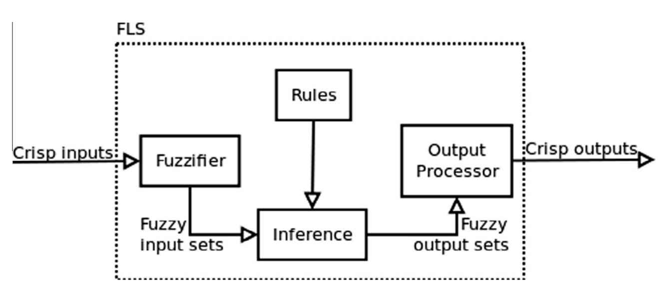

Fig. 1. Block diagram describing a T1FLS.