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Approaches to manage hesitant fuzzy linguistic information based on the cosine distance and similarity measures for HFLTSs and their application in qualitative decision making

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ABSTRACT

Qualitative and hesitant information is common in practical decision making process. In such complicated decision making problem, it is flexible for experts to use comparative linguistic expressions to express their opinions since the linguistic expressions are much closer than single or simple linguistic term to human way of thinking and cognition. The hesitant fuzzy linguistic term set (HFLTS) turns out to be a powerful tool in representing and eliciting the comparative linguistic expressions. In order to develop some approaches to decision making with hesitant fuzzy linguistic information, in this paper, we firstly introduce a family of novel distance and similarity measures for HFLTSs, such as the cosine distance and similarity measures, the weighted cosine distance and similarity measures, the order weighted cosine distance and similarity measures, and the continuous cosine distance and similarity measures. All these distance and similarity measures are proposed from the geometric point of view while the existing distance and similarity measures over HFLTSs are based on the different forms of algebra distance measures. Afterwards, based on the hesitant fuzzy linguistic cosine distance measures between hesitant fuzzy linguistic elements, the cosine-distance-based HFL-TOPSIS method and the cosine-distance-based HFL-VIKOR method are developed to dealing with hesitant fuzzy linguistic multiple criteria decision making problems. The step by step algorithms of these two methods are given for the convenience of applications. Finally, a numerical example concerning the selection of ERP systems is given to illustrate the validation and efficiency of the proposed methods.

1. Introduction

Hesitant fuzzy set (HFS, Torra, 2010) and hesitant fuzzy linguistic term set (HFLTS, Rodríguez, Martínez, & Herrera, 2012) are two tools to represent imprecise and hesitant information. The motivation for introducing these two types of sets is that it is sometimes difficult to determine the membership degree of an element to a set due to the doubt among different values. The difference between HFS and HFLTS is that the HFS is used to represent quantitative hesitant information, while the HFLTS is proposed to denote qualitative vague information. Recently, both of these two sets have been investigated by many scholars (Readers who have more interest, please refer to the special issue in *International Journal of Intelligent Systems, vol. 29, No. 6*). In this paper, we focus on the HFLTS.

Qualitative information exists everywhere in our daily life as it is close to human's subjective cognition. In 1975, Zadeh (1975) proposed the fuzzy linguistic approach to represent qualitative information as linguistic variables, which opens a completely new research area named computing with words (CWW). In the framework of classical CWW, the values of a linguistic variable are represented by single or simple linguistic terms. However, this is not adequate to represent human's more comprehensive cognition. For example, the linguistic term can only express "*high*" speed, but can not represent "*a little high*" speed or "*lower than high*" speed. In many real-life situations, it is more flexible for experts to express their opinions by linguistic expressions but not just one linguistic term. The introduction of HFLTS solves this problem perfectly. The HFLTS is defined as an ordered finite subset of the consecutive linguistic terms (Rodríguez et al., 2012). With HFLTS, the linguistic information, especially the linguistic expressions, can be elicited by using HFLTS and context-free grammars. That is to say, with the HFLTS, we can compute with linguistic

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expressions directly. Hence, the introduction of HFLTS is a worthwhile contribution to the theory of CWW.

As the HFLTS increases the flexibility and capability of the elicitation of linguistic information by means of linguistic expressions, it has attracted more and more scholars' attention and many fruitful achievements have been proposed. After introducing the concept of HFLTS and its basic operations and properties, Rodríguez et al. (2012) developed a multi-criteria linguistic decision making model with linguistic expressions. Later on, they (Rodríguez, Martínez, & Herrera, 2013) further proposed a linguistic group decision making model which deals with comparative linguistic expressions in the process of group decision making. To make it much easier to understand, Liao, Xu, Zeng, and Merigó (2015c) gave the mathematical definition of HFLTS and introduced the hesitant fuzzy linguistic element (HFLE) to represent the hesitant fuzzy linguistic value of a linguistic variable. Liao, Xu, and Zeng (2014b), Liao et al. (2015c) also introduced different kinds of distance measures, similarity measures and correlation coefficients for HFLTSs. Wei, Zhao, and Tang (2014) investigated the comparison and aggregation methodologies for HFLTSs. Based on the fuzzy envelope, a new representation of the HFLTS was introduced by Liu and Rodríguez (2014). In order to apply the HFLTS into decision making process, motivated by the hesitant fuzzy preference relation (Liao, Xu, & Xia, 2014a), Zhu and Xu (2014) defined the hesitant fuzzy linguistic preference relation (HFLPR) and investigated its consistency. Liu, Cai, and Jiang (2014) further investigated the additive consistency of a HFLPR. Some decision making approaches were also extended into HFLTS circumstances. For example, for the multi-criteria decision making (MCDM) problem in which the opinions of an expert are represented by HFLTSs, Beg and Rashid (2013) proposed a TOPSIS-based method to solve it. As to the hesitant fuzzy linguistic MCDM problem where some criteria conflict with each other, Liao, Xu, and Zeng (2015a) established a HFL-VIKOR method and implemented it into practical decision making processes.

All the above-mentioned results show that the HFLTS is a good research topic in the field of decision making. However, as the HFLTS is just introduced in 2012, the foundation of this theory is not strong enough and thus much work needs to be done to fill this gap. Roughly speaking, the achievements on HFLTSs in the literature can be classified into the following parts:

- Basic definitions and operations over HFLTSs (see Liao et al. (2015c), Liu et al. (2014), Liu & Rodríguez (2014), Rodríguez et al. (2012), Zhu & Xu (2014));
- Information fusion methods with HFLTSs, such as different forms of aggregation operators (see Rodríguez et al. (2012), Wei et al. (2014));
- Measures of HFLTSs, including the correlation measures (Liao et al., 2015c), the distance measures, and the similarity measures (Liao et al., 2014b);
- Distinct decision making methods, such as the group decision making method (Rodríguez et al., 2013), the HFL-TOPSIS method (Beg & Rashid, 2013; Liu & Rodríguez, 2014), and the HFL-VIKOR method (Liao et al., 2015a).

Since both the HFL-TOPSIS method and the HFL-VIKOR method are based on the distance measures of HFLTSs, in this paper, we focus our attention on the distance and similarity measures of HFLTSs. The basic principle of the TOPSIS method is to find an alternative which has the shortest distance from the positive-ideal solution and the furthest distance from the negative-ideal solution (Beg & Rashid, 2013; Hwang & Yoon, 1981); while the main idea of VIKOR method is to determine a compromise solution, which provides a maximum "*group utility*" for the "*majority*" and a minimum "*individual regret*" for the "*opponent*", for a MCDM problem with non-commensurable and conflicting criteria by mutual concessions (Opricovic & Tzeng, 2004), but such "*group utility*" and "*individual regret*" are measured by the distances measures of HFLTSs. Hence, the distance measures of the HFLTSs play a very important role in these two methods. As we can see from Liao et al. (2014b) that all the distance measures proposed by them were based on the different forms of algebra distance measures, such as the Hamming distance measure, the Euclidean distance measure and the Hausdorff distance measure. In this paper, we try to propose some novel distance measures which are not based on the algebraic distance measures but from the geometric point of view. A sort of cosine distance and similarity measures are introduced for HFLTSs, based on which, the cosine-distance-based HFL-TOPSIS and the cosine-distance-based HFL-VIKOR method are further established. A numerical example concerning the selection of ERP system is given to illustrate the validation and efficiency of the proposed method.

The remainder of this paper is organized as follows: Section 2 reviews the concepts of HFLTS and the distance and similarity measures. Section 3 proposes different forms of cosine distance and similarity measures for HFLTSs. The cosine-distance-based HFL-TOPSIS method and the cosine-distance-based HFL-VIKOR method are developed in Section 4. A numerical example is given in Section 5 to show the applicability and validation of the methodologies. The paper ends with some concluding remarks in Section 6.

2. Hesitant fuzzy linguistic term set and the distance and similarity measures

2.1. Hesitant fuzzy linguistic term set

In traditional fuzzy linguistic approach for qualitative decision making, experts are supposed to use single linguistic term to represent the value of a linguistic variable. However, this is not appropriate to tackle more complicated decision making problems as in many cases, the experts can not give their assessments in single terms but linguistic expressions. For example, when evaluating the performance of an operation system, one engineer may say "its performance is *between medium and high*", while the other may deem "it is *at least a little high*". As traditional fuzzy linguistic approach can only use single term, such as "*medium*", "*high*" or "*a little high*", to express the cognition of a person, in order to represent comprehensive linguistic expressions, Rodríguez et al. (2012) introduced the concept of hesitant fuzzy linguistic term set, which can be used to elicit several linguistic terms for a linguistic variable.

Definition 1 Rodríguez et al., 2012. Let $S = \{s_0, \ldots, s_\tau\}$ be a linguistic term set. A hesitant fuzzy linguistic term set (HFLTS), H_S, is an ordered finite subset of the consecutive linguistic terms of S.

Example 1. Here we just consider a simple example that an expert evaluates the operational complexity of three automatic systems, represented as $x_1, x_2, and x_3$. Since this criterion is qualitative, it is impossible to give crisp values but only linguistic terms. The operational complexity of these automatic systems can be taken as a linguistic variable. The linguistic term set of the operational complexity can be set up as:

$$S = \{s_{-3} = very\ complex,\ s_{-2} = complex, s_{-1} = a\ little\ complex,$$
$$s_0 = medium,\ s_1 = a\ little\ easy,\ s_2 = easy,\ s_3 = very\ easy\}$$