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Path relinking for the fixed spectrum frequency assignment problem

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ABSTRACT

The fixed spectrum frequency assignment problem (FS-FAP) is a highly relevant application in modern wireless systems. This paper presents the first path relinking (PR) approach for solving FS-FAP. We devise four relinking operators to generate intermediate solutions (or paths) and a tabu search procedure for local optimization. We also adopt a diversity-and-quality technique to maintain population diversity. To show the effectiveness of the proposed approach, we present computational results on the set of 42 benchmark instances commonly used in the literature and compare them with the current best results obtained by any other existing methods. By showing improved best results (new upper bounds) for 19 instances, we demonstrate the effectiveness of the proposed PR approach. We investigate the impact of the relinking operators and the population updating strategy. The ideas of the proposed could be applicable to other frequency assignment problems and search problems.

1. Introduction

Frequency assignment represents a key issue in wireless communications systems and has attracted a lot of attentions (Audhya et al., 2013; Castelino, Hurley, & Stephens, 1996; Fischettti, Lepsch, Minerva, Romanin-Jacur, & Toto, 2000; Flood & Allen, 2013;Hale, 1980; Hao, Dorne, & Galinier, 1998; Hurley, Smith, & Thiel, 1997; Kim, Smith, & Lee, 2007; Král, 2005; Lai & Coghill, 1996; Maniezzo & Carbonaro, 2000; San José-Revuelta, 2007; Smith, Hurley, & Thiel, 1998; Subramanian, Gupta, Das, & Jing, 2008; Tiourine, Hurkens, & Lenstra, 2000; Wang & Rushforth, 1996; Wu, Dai, & Zhao, 2013). The fixed spectrum frequency assignment problem (FS-FAP) studied in this study is among the most typical frequency assignment problems (Aardal, Van Hoesel, Koster, Mannino, & Sassano, 2007; Eisenblätter & Koster, 2010; Montemanni & Smith, 2010).

Informally, given a radio network, FS-FAP aims to assign a set of available frequencies to each transmitter of the network such that the level of interference of the frequency assignment is minimized. Technically, interference occurs when two close transmitters are assigned frequencies which are close in the frequency spectrum.

From a graph-theoretic point of view, FS-FAP can be defined as follows (Aardal et al., 2007; Montemanni & Smith, 2010). Given a double weight undirected graph $G = (V, E, D, P)$ with a vertex set V, an edge set E, and two edge weight sets D and P, as well as a set F of consecutive frequencies, FS-FAP is to find a mapping (i.e., a frequency assignment) f from V to F, (i.e., $f : V \rightarrow F$) such that an objective or cost function is minimized. In the present model, a vertex $v \in V$ corresponds to a transmitter of the network, an edge $e \in E$ represents a pair of transmitters for which the assigned frequencies are constrained, D is the set of edge weights (d_{ij}, the first type) defining the separation constraints between frequencies of any two adjacent vertices, and P is the set of edge weights (p_{ij}, the second type) representing the penalty values measuring the degree of interference defined as follows. Given a frequency assignment f and an edge $e(i,j) \in E$, the penalty p_{ij} is incurred if $|f(i) - f(j)| \leqslant d_{ij}$, i.e., if the separation constraint is violated. Then the cost of the frequency assignment f is defined as the total interference generated by f which is given by the summation of the incurred penalty values (p_{ij}). The goal of FS-FAP is then to determine a frequency assignment of minimum cost. Note that in the literature, there exist other similar FS-FAP models which differ mainly in the way the cost function is defined (Aardal et al., 2007; Hale, 1980; Hao et al., 1998; Hurley et al., 1997; Kendall et al., 2004; Mabed, Caminada, & Hao, 2011; Park, Kim, & Moon, 2002).

Frequency assignment problems like FS-FAP are closely related to the bandwidth coloring problem (also called the restricted T-coloring problem) with k fixed colors (Dorne & Hao, 1998, chap. 6; Jin & Hao, 2014; Johnson, Mehrotra, & Trick, 2008; Lai & Lü, 2013; Malaguti & Toth, 2008; Roberts, 1991) and known to be computationally hard in general. Indeed, the FS-FAP problem can be shown to be NP-hard since it is reduced to the NP-hard graph k-coloring problem (Hale, 1980). Given the high complexity of FS-FAP, considerable efforts have been made to develop effective

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heuristic methods. These include single solution based approaches like tabu search (Castelino et al., 1996; Hao et al., 1998; Montemanni, Moon, & Smith, 2003), adaptive local search (Kendall et al., 2004), heuristic manipulation technique (Montemanni & Smith, 2010). Population-based approaches are also very popular: ANTS algorithm (Maniezzo & Carbonaro, 2000), genetic algorithms (Dorne & Hao, 1996; Park et al., 2002; San José-Revuelta, 2007), hybrid evolutionary algorithms (Dorne & Hao, 1995; Jin, Wu, Horng, & Tsai, 2001; Kim et al., 2007; Lai & Coghill, 1996), multiple strategy method (Hurley et al., 1997). In addition to heuristic methods, approaches using integer programming were also proposed in the literature to determine lower bounds for the FS-FAP problem, such as those in Montemanni, Smith, and Allen (2001, 2004).

Recently, the population-based path relinking approach (Glover, 1997; Glover, Laguna, & Martí, 2000) has attracted special attention in combinatorial optimization, and shows outstanding performances in solving a number of difficult problems, such as antibandwidth problem (Duarte, Martí, Resende, & Silva, 2011), bandwidth coloring problem (Lai, Lü, Hao, Glover, & Xu, 2014), clustering (Martins de Oliveira, Nogueira Lorena, Chaves, & Mauri, 2014), flow shop sequencing and scheduling (Costa, Goldbarg, & Goldbarg, 2012; Reeves & Yamada, 1998), unconstrained binary quadratic optimization (Wang, Lü, Glover, & Hao, 2012), and web services composition (Parejo, Segura, Fernandez, & Ruiz-Cortés, 2014). In this paper, we introduce an effective path relinking algorithm for solving FS-FAP with the following main contributions.

From the perspective of algorithm design, we base our approach on the general path relinking framework and devise dedicated relinking operators. These operators are used to create solution paths from one initiating solution to a guiding solution. For the purpose of local optimization, we develop a specific tabu search procedure. To maintain a healthy diversity of the solution population, we apply a quality-and-diversity technique to update the solution pool. The integration of these ingredients leads to an effective algorithm.

Indeed, the proposed algorithm is assessed on the set of 42 benchmark instances commonly used in the literature and shows a very competitive performance in comparison with the best performing method dedicated to the studied problem in the literature. In particular, the proposed algorithm is able to discover improved best known results (new upper bounds) for 19 instances and matches the best known results for 21 other cases.

The rest of this paper is organized as follows. In Section 2, we present the proposed algorithm. In Section 3, we show computational results by comparing them with the results of the best performing algorithm in the literature. In Section 4, we investigate some key ingredients of the proposed algorithm. In Section 5, we draw conclusions and discuss several research perspectives.

2. Path relinking algorithm for FS-FAP

The path relinking algorithm presented in this paper is a hybrid population algorithm that combines four essential ingredients: population initialization, local optimization method, relinking operator, and population updating strategy. In this section, we present the general path relinking algorithm and its ingredients.

2.1. Search space and evaluation function

We first define the search space Ω explored by the algorithm and the evaluation function which is used to measure the quality of a candidate solution.

A solution of FS-FAP is a mapping from V to F ($f: V \rightarrow F$), thus a solution s can be represented by a one-dimensional vector where each variable represents a vertex and its value is the assigned frequency. The search space Ω explored by the algorithm is composed of all possible assignment, and hence the size of the search space is bounded by $O(|F|^{|V|})$.

To evaluate the quality of a frequency assignment $s = (f_1, f_2, \ldots, f_n) \in \Omega$, we calculate the degree of interference as follows:

$$Cost(s) = \sum_{e(i,j)\in E: |f_i - f_j| \leqslant d_{ij}} p_{ij} \qquad (1)$$

where $d_{ij} \in D$ represents the distance constraint between the frequencies of two adjacent vertices i and j, and p_{ij} represents the resulting penalty when the distance constraint is violated, i.e., $|f_i - f_j| > d_{ij}$. This cost function (to be minimized) is also referred as the objective function in the rest of the paper.

2.2. Main framework

Algorithm 1. Pseudo-code of our path relinking algorithm for FS-FAP

```
1: Input: Problem instance $I$, the size of population $p$
2: Output: the best solution $s^*$ found
3: repeat
4:   $P = \{s^1, \ldots, s^p\} \leftarrow$ Population_Initialization $(I, p)$   /* Section 2.3 */
5:   if it is not in the first loop then
6:     $s^w = arg\ max\{f(s^i) : i = 1, \ldots, p\}$
7:     $P \leftarrow P \cup \{s^*\} \setminus \{s^w\}$
8:   end if
9:   $s^* = arg\ min\{f(s^i) : i = 1, \ldots, p\}$
10:    $PairSet \leftarrow \{(s^i, s^j) : 1 \leqslant i < j \leqslant p\}$
11:    while $PairSet \neq \emptyset$ do
12:      Randomly pick a solution pair $(s^i, s^j) \in PairSet$
13:      $PairSet \leftarrow PairSet \setminus \{(s^i, s^j)\}$
14:      $s' \leftarrow PathRelinking(s^i, s^j)$, $s'' \leftarrow PathRelinking(s^j, s^i)$ /* Section 2.5 */
15:      $s' \leftarrow TabuSearch(s')$   /* Section 2.4 */
16:      if $f(s') < f(s^*)$ then
17:        $s^* \leftarrow s'$, $f(s^*) \leftarrow f(s')$
18:      end if
19:      $P \leftarrow UpdatingPool(PairSet, P, s')$
20:      $PairSet \leftarrow UpdatingPool(PairSet, P, s')$   /* Section 2.6 */
21:      $s'' \leftarrow TabuSearch(s'')$   /* Section 2.4 */
22:      if $f(s'') < f(s^*)$ then
23:        $s^* \leftarrow s''$, $f(s^*) \leftarrow f(s'')$
24:      end if
25:      $P \leftarrow UpdatingPool(PairSet, P, s'')$
26:      $PairSet \leftarrow UpdatingPool(PairSet, P, s'')$   /* Section 2.6 */
27:    end while
28: until a stop criterion is met
```

The general procedure of our path relinking algorithm is shown in Algorithm 1, where the following notations are used. s^* and s^w respectively represent the best solution found so far and the worst solution in the population, and *PairSet* is the set of solution pairs (s^i, s^j) and is initially composed of all possible solution pairs (s^i, s^j) in the population.

The proposed algorithm starts with an initial population P (line 4) which includes p different solutions, where each of them is