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A 2-phase constructive algorithm for cumulative vehicle routing problems with limited duration

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ABSTRACT

The Clarke & Wright (C&W) algorithm is one of the most widely used classical heuristics in capacitated Vehicle Routing Problems (VRPs) in which a linear function of distance is considered as the objective function. The C&W algorithm is very simple and easy to implement, and produces fairly good solutions very fast. In this study, the C&W algorithm is adopted for the cumulative VRP with limited duration (CumVRP-LD) where load is also considered in the objective function as well as distance. The most common applications of cumulative VRPs are the determination of routing policies that minimize total fuel consumption. A 2-phase constructive heuristic approach including the K-means clustering algorithm is proposed to improve the computational performance of the modified C&W algorithm for CumVRP-LD. The main contribution of this study is the definition of a new extended formulation that captures truck-load and travel distance by considering the unique characteristics of the problem and to develop a fast and easy implemented constructive algorithm for CumVRP-LD. Such approaches are necessary for the development of systems that respond fast, possibly online, to changes in the real problem situations.

1. Introduction

The Vehicle Routing Problem (VRP) is one of the most widely investigated problems in the optimization literature. It is an attractive subject to investigate, especially for companies with transportation operations. The classical VRP designs set of routes for *m* identical vehicles which have to serve *n* customers. Each route starts and ends at a depot and each customer can be served by only one vehicle. One of the most well known versions of routing problems is the capacitated VRP in which each vehicle has a capacity *Q* that cannot be exceeded during traveling. A detailed description for several variations of VRPs can be obtained in Toth and Vigo (2008) and Golden, Raghavan, and Wasil (2008).

Since VRPs are NP-hard problems, big size instances cannot be solved exactly by polynomial time algorithms (Lenstra & Rinnooy Kan, 1981). Because of the inadequacy of exact algorithms, various heuristics and metaheuristics have been developed to obtain good VRP solution in reasonable computational time for big instances. Classical heuristics are very simple and easy to implement. Moreover they can obtain fairly good solutions in a very short time. On the other hand, they have limited exploration ability compared with metaheuristics (Altınel & Öncan, 2005). Readers are referred to Laporte, Gendreau, Potvin, and Semet (2000) for a review on heuristics developed for VRPs.

The Clarke & Wright (C&W) algorithm (Clarke & Wright, 1964) is one of the well known and widely used heuristics for capacitated VRPs with distance minimization because of its simplicity and high speed (Altınel & Öncan, 2005). It is also known as a *savings* algorithm that uses savings criteria to schedule the customers on the tours. Several enhancements for the C&W algorithm have been developed in the literature (Altınel & Öncan, 2005; Gaskell, 1967; Golden, Magnanti, & Nguyen, 1977; Nelson, Nygard, Griffin, & Shreve, 1985; Paessens, 1988). Almost all of these algorithms use distance values to compute saving. To the best of the authors' knowledge, only Altınel and Öncan (2005) considered the demand of customers in the computation of saving. The load of vehicles has not been included in the scope of this study.

Additionally, in this study, load flow is also considered in the objective function of a VRP as well as distance information. Since load flow can be represented as a step function, Kara, Kara, and Yetiş (2008) named routing problems with flow based cost functions as *cumulative routing problems*. In the delivery case, the moved load of a vehicle decreases as a step function while the customers are being visited. The reverse relation is valid for the collection case that total load increases as a step function

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towards the end of the tour. In this study, the time limitation is also taken into account for each vehicle. Every vehicle traveling through a tour cannot exceed a predetermined time limit which makes the problem more realistic. Because of the time limit constraints and the objective function, the problem investigated in this study is referred to CumVRP-LD, where LD stands for *Limited Duration*. Energy and CO_2 emissions minimization VRP problems are among the real life applications of VRPs with flow based objective functions.

The main contributions of this paper are (1) a new expression for the savings function that considers the load of vehicles, (2) a hybrid constructive algorithm, which incorporates the C&W algorithm and K-means clustering, to obtain fairly good solutions in short computational time for the CumVRP-LD. Such approaches are necessary for the development of systems that respond fast, possibly online, to changes in the real problem situations. Furthermore, it sets the foundation for the study of special structures that allow faster convergence to the optimal solution.

The paper is organized as follows. In the next section, a brief explanation and a MIP formulation for CumVRP-LD is presented. Literature review on various VRPs with fuel consumption minimization and the C&W algorithm enhancements is presented in Section 3. Constructive algorithms developed for CumVRP-LD are explained in Section 4. Computational results are given in Section 5 and concluding remarks and suggestions for further study are presented in Section 6.

2. Problem statement and formulation

A VRP can be stated on a directed graph $G = (V, A)$ where $V = \{0, 1, \dots, N, N+1\}$ is the set of vertices (customers) and $A = \{(i, j) \mid i, j \in V, i \neq j\}$ is the set of arcs. Vertices 0 and $N+1$ refer to the depot, where each vehicle starts from 0 and finishes at $N+1$ for the corresponding route. There is a set of $M = \{1, \dots, K\}$ referring to the vehicles that are used to deliver orders to the customers. Each customer can be served only once and all customers must be served. The distance from customer i to customer j is represented by d_{ij} and each customer has demand c_i. Each vehicle can perform only one tour. Total traveling time of a vehicle cannot exceed the predetermined working time and each vehicle has its own capacity that total demand of the customers in the same tour cannot exceed it. An instance should satisfy the following inequalities to be feasible:

$$(d_{0i} + d_{i0})/v \leq TL \qquad \forall i \in V \setminus \{0\} \tag{1}$$

$$c_i \leq Q_k \qquad \forall i \in V \setminus \{0\}, \forall k \tag{2}$$

where v is the velocity, TL is the time limit that a vehicle can operate, and Q_k is the capacity of vehicle k.

Since VRPs are NP-hard problems, they cannot be solved exactly by polynomial time algorithms (Lenstra & Rinnooy Kan, 1981). CumVRP-LD is a more complicated variation of VRP. Therefore, it is NP-hard in a strong sense and exact optimization methods may be unable to solve large instances in acceptable CPU times.

Most of the VRP studies have objective functions that are linear functions of distance (Li, 2012), such as total travel time or total transportation cost. In this study, total cost, which is a function of both load and distance, is minimized. A MIP formulation for CumVRP-LD is given as follows:

$$\min \sum_{i=0}^{N} \sum_{j=1}^{N+1} \sum_{k=1}^{K} d_{ij}(a_k x_{ijk} + b_k q_{ijk}) \tag{3}$$

$$\sum_{i=0, i\neq j}^{N} \sum_{k=1}^{K} x_{ijk} = 1 \qquad j = 1, \dots, N \tag{4}$$

$$\sum_{i=0, i\neq j}^{N} x_{ijk} = \sum_{i=1, i\neq j}^{N+1} x_{jik} \qquad j = 1, \dots, N, \forall k \tag{5}$$

$$\sum_{j=1}^{N} x_{0jk} \leq 1 \qquad \forall k \tag{6}$$

$$\sum_{i=0}^{N} \sum_{j=1}^{N+1} \left(\frac{d_{ij}}{v} + p_j \right) x_{ijk} \leq TL \qquad \forall k \tag{7}$$

$$\sum_{i=0, i\neq j}^{N} q_{ijk} - \sum_{i=1, i\neq j}^{N+1} q_{jik} = c_j \sum_{i=0, i\neq j}^{n} x_{ijk} \qquad i = 1, \dots, N, \forall k \tag{8}$$

$$\sum_{i=0, i\neq j}^{N} q_{ijk} - \sum_{h=1}^{N+1} c_h x_{jhk} \geq c_j \sum_{i=0, i\neq j}^{N} x_{ijk} \qquad j = 1, \dots, N, \forall k \tag{9}$$

$$q_{ijk} \leq (Q_k - c_i) x_{ijk} \qquad i = 0, \dots, N, j = 1, \dots, N, i \neq j, \forall k \tag{10}$$

$$q_{ijk} = 0 \qquad i = 1, \dots, N, j = N+1, \forall k \tag{11}$$

$$x_{ijk} \in \{0, 1\} \qquad i = 0, \dots N, j = 1, \dots N+1, \forall k \tag{12}$$

$$q_{ijk} \geq 0 \qquad i = 0, \dots N, j = 1, \dots N+1, \forall k \in K \tag{13}$$

The notation used for the MIP model is given in Table 1. This model is an extended version of the model proposed by Kara, Kara, and Yetiş (2007), with time limitation constraints and heterogeneous fleet type. The objective given by (3) is a function of load and distance where a_k and b_k are coefficients for vehicle k. Constraints (4) guarantee that each customer should be served. The flow conservation for each customer is satisfied by constraints (5), i.e. if a vehicle arrives at a customer to serve, it should also leave that customer. Constraints (6) ensure that each vehicle k can perform at most one trip. If these constraints are relaxed, the problem becomes a cumulative multi-trip VRP with time limitation (Cinar, Gakis, & Pardalos, 2015). A time limit which cannot be exceeded by any vehicle is satisfied by constraints (7). Subtour elimination constraints are implied by (8). Only delivery was taken into account, reloading was not considered in the problem. The difference between the load of a vehicle arriving to a customer and the load departing from that customer is equal to the demand of the customer. Constraints (9–11) ensure valid capacity constraints for the CumVRP-LD. If a customer j is assigned to a vehicle k, then it may be the first, the last or an intermediate customer

Table 1
Notation for MIP.

Sets:	
V	Set of customers, $V = \{0, 1, \dots, N, N+1\}$, where 0 and $N+1$ refer to the depot
M	Set of vehicles $M = \{1, \dots, K\}$
Decision variables:	
q_{ijk}	Total load transported from customer i to customer j by vehicle k for $i \in V \setminus \{n+1\}, j \in V \setminus \{0\}, k \in K$
x_{ijk}	1 if vehicle k visits customer j immediately after customer i, 0 o.w. for $i \in V \setminus \{n+1\}, j \in V \setminus \{0\}, k \in K$
Parameters:	
TL	The time limit that a vehicle can operate
v	Average speed of a vehicle
p_i	Service time for customer $i (p_0 = p_{N+1} = 0)$
d_{ij}	Distance between customers i and j
c_i	Demand of customer i
Q_k	Capacity of vehicle k
a_k	Coefficient for vehicle k
b_k	Coefficient for load of vehicle k