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A new interval-valued knowledge measure for interval-valued intuitionistic fuzzy sets and application in decision making

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ABSTRACT

In this paper, firstly we discuss some entropy measures for the interval-valued intuitionistic fuzzy sets (IvIFSs). Then we extend the knowledge measure for the intuitionistic fuzzy sets (IFSs) to propose a new interval-valued knowledge measure for the IvIFSs. Based on the proposed knowledge measure we construct a new interval-valued information entropy measure for IvIFSs, which is an extended notion of the entropy measures for IFSs. The proposed knowledge measure is defined as an interval of amounts of knowledge measured on an IvIFS, related to the uncertain information in terms of interval membership degree and interval non-membership degree. In comparison with other existing measures, it seems to be simpler and more intuitively appealing. Several illustrative examples are performed to demonstrate the effectiveness and practicality of the proposed method in handling with the increasing complexity of the decision making problems.

1. Introduction

A generalization of the notion of intuitionistic fuzzy set (IFS), proposed by Atanassov and Gargov (1989), is the so-called interval-valued intuitionistic fuzzy set (IvIFS). The justification of this generalization is that the degrees of membership or non-membership are sometimes assumed not exactly as a number but as a whole interval, which introduces an additional uncertainty to IFSs. As an important topic in the theory of fuzzy sets, many entropy measures of IFSs have been investigated widely by many researchers from different views. Due to the increasing complexity of the real-life decision making problems and the lack of precise information about the problem domains, the IvIFSs may be more suitable to represent uncertain decision information. Therefore, it is highly necessary and significant to construct a reliable entropy measure for IvIFSs.

At present there are several measures for IvIFSs and their applications in various areas such as mathematics, physics, statistics, engineering, social sciences and many others. Atanassov and Gargov (1989) introduced IvIFSs with several properties and applications. Xu (2007) proposed some methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making. Ahn, Han, Oh, and Lee (2011)) gave an application on medical diagnosis using IvIFSs. The entropy measure of IvIFSs has been applied widely in decision making problems. Ye (2010) proposed two entropy measures for IvIFSs and established an entropy weight model for determining the criteria weights on alternatives. Zhang, Ma, Su, and Zhang (2011) ensured that their proposed entropy measure of IvIFSs complies with the extended form of De Luca and Termini's axioms for fuzzy entropy. Wei, Wang, and Zhang (2011) proposed an entropy measure for IvIFSs and its application to solve problems on multi-criteria fuzzy decision making. Chen, Yang, Wang, and Yue (2013) proposed an entropy weight model using cotangent function to determine the entropy weights for fuzzy multi-criteria group decision-making (FMCGDM) problems. Wei and Zhang (2015) proposed two entropy measures based on cosine function and applied them to assess the experts' weights and to solve multi criteria fuzzy group decision-making problems.

Although there exists several entropy measures for IFSs and IvIFSs, many unreasonable cases can be made by such measures (Szmidt & Kacprzyk, 2007, Ye, 2010, Zhang et al., 2011). It is worth noting that the uncertainties involving the intervals of membership and non-membership degrees of IvIFSs imply additional uncertainty in the measures of IvIFSs. It is therefore reasonable to model the fuzzy measures for IvIFSs as intervals, which have been recently proposed by some researchers (Nguyen & Kreinovich, 2008, Stachowiak, Zywica, Dyczkowski, & Wojtowicz, 2015). Moreover, Zhai and Mendel (2011) mentioned a fundamental principle to guide the use of uncertainty measures as follows: "The principle of maximum uncertainty, which states that a conclusion should maximize the relevant uncertainty within constraints given by the verified premises". Inspired of that, in this paper we extend the knowledge measure for IFSs developed by Nguyen (2015), to present

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a novel interval-valued knowledge measure for IvIFSs. Based on the proposed knowledge measure, we construct a new information interval-valued entropy measure for IvIFSs. Several illustrative examples are performed to assess how reasonable and accurate the new measure is in comparison with others.

The remainder of the paper is organized as follows: in Section 2 we review some basic concepts and axiomatic properties of entropy measures for IvIFSs. Section 3 provides a review and discussion on the existing measures for IvIFSs. Section 4 develops a new interval-valued knowledge measure and information entropy measure for IvIFSs. Section 5 gives some comprehensive comparative examples. Section 6 shows some applications of the proposed approach. Section 7 summarizes and discusses on obtained results. Finally, Section 8 provides concluding remarks.

2. Preliminaries

2.1. Interval-valued Intuitionistic Fuzzy Sets

Atanassov and Gargov (1989) generalized the concept of interval-valued fuzzy sets (IvFSs) and IFSs by introducing an interval-valued intuitionistic fuzzy set (IvIFS) A^I over a finite universe of discourse $X = \{x_1,\ x_2, \ldots, x_n\}$ as:

$$A^I = \{\langle x_i, M_{A^I}(x_i),\ N_{A^I}(x_i)\rangle | x_i \in X\} \tag{1}$$

where $M_{A^I}(x_i) = [\mu^-_{A^I}(x_i), \mu^+_{A^I}(x_i)] \subset [0, 1]$ denotes the interval membership degree and $N_{A^I}(x_i) = [\nu^-_{A^I}(x_i), \nu^+_{A^I}(x_i)] \subset [0, 1]$ denotes the interval non-membership degree of an element x_i to IvIFS A^I, such that for every $x_i \in X$, $\mu^+_{A^I}(x_i) +\ \nu^+_{A^I}(x_i) \le 1$ and

$$\left[\pi^-_{A^I}(x_i), \pi^+_{A^I}(x_i)\right] = \left[1 - \mu^+_{A^I}(x_i) - \nu^+_{A^I}(x_i),\ \ 1 - \mu^-_{AA^I}(x_i) - \nu^-_{A^I}(x_i)\right], \tag{2}$$

where $[\pi^-_{A^I}(x_i), \pi^+_{A^I}(x_i)]$ denotes the interval hesitancy degree of x_i to A^I. The complementary set A^I_C of A^I is defined as: $A^I_C = \{\langle x_i,\ N_{A^I}(x_i), M_{A^I}(x_i)\rangle | x_i \in X\}$.

2.2. Axiomatic properties of entropy measures for IvIFSs

De Luca and Termini (1972) firstly introduced the non-probabilistic entropy for fuzzy sets (FSs), which was regarded as a measure of a quantity of information, average intrinsic information received from objects described by means of FSs. Later on, Szmidt and Kacprzyk (2001) extended this function “entropy” and its axiomatic properties into the Atanassov's IFSs. Based on Szmidt and Kacprzyk's axioms, Sun and Liu (2012) introduced the following properties of entropy for IvIFSs.

Let X be a universe of discourse, for any IvIFS given by $A^I = \{\langle x_i, M_{A^I}(x_i),\ N_{A^I}(x_i)\rangle | x_i \in X\}$, the entropy function $E(A^I)$ of A^I should satisfy the following (Sun & Liu, 2012): $\forall x_i \in X$,

(R1.1) $E(A^I) = 0$ *iff* A^I is a crisp set;
(R1.2) $E(A^I) = 1$ *iff* $M_{A^I}(x_i) =\ N_{A^I}(x_i)$;
(R1.3) $E(A^I) \le E(B^I)$ if A^I is less fuzzy then B^I, i.e.

$$M_{A^I}(x_i) \le M_{B^I}(x_i) \text{ and } N_{A^I}(x_i) \ge N_{B^I}(x_i) \quad \text{for } M_{B^I}(x_i) \le N_{B^I}(x_i)$$

$$\text{or } M_{A^I}(x_i) \ge M_{B^I}(x_i) \text{ and } N_{A^I}(x_i) \le N_{B^I}(x_i) \quad \text{for } M_{B^I}(x_i) \ge N_{B^I}(x_i)$$

(R1.4) $E(A^I) = E(A^I_C)$, A^I_C is a complement of A^I.

Sun and Liu (2012) pointed out unreasonable cases of entropy measures for IvIFSs from Zhang, Jiang, Jia, & Luo (2010), Wei et al. (2011) and Zhang et al. (2011). As it will be shown later in examples of Section 4, their proposed measure is also unreasonable in discriminating the cases when $M_{A^I}(x_i) =\ N_{A^I}(x_i)$. Chen et al. (2013) proved limitation of entropy measures from Ye (2010). Their proposed cotangent entropy measure is also unreasonable in discriminating the cases when $M_{A^I}(x_i) =\ N_{A^I}(x_i)$. Besides, based on distance, Zhang, Xing, Liu, Ye, and Tang (2014) proposed the axiomatic properties of entropy measure for IvIFSs as follows: $\forall x_i \in X$,

(R2.1) $E(A^I) = 0$ *iff* A^I is a crisp set;
(R2.2) $E(A^I) = 1$ *iff* $M_{A^I}(x_i) =\ N_{A^I}(x_i) = [0.5,\ 0.5]$;
(R2.3) $E(A^I) \le E(B^I)$ if $d(A^I, G) \ge d(B^I, G)$, $\forall A^I, B^I \in IvIF(X)$, where $G = 0.5,\ 0.5$;
(R2.4) $E(A^I) = E(A^I_C)$, A^I_C is a complement of A^I.

Entropy is often viewed as a dual measure of the amount of knowledge. Various well-known entropy measures for IFSs had been also presented by many scholars, such as De Luca and Termini (1972), Szmidt and Kacprzyk (2001, 2014), Wang (2012) and Zhang (2013). However, Szmidt and Kacprzyk (2007) have found some problems with existing entropy measures. Nevertheless, the entropy measures proposed by Szmidt and Kacprzyk (2014) have problems with distinguishing the IFSs when $\mu_A = \nu_A$. The reason of these drawbacks is that the existing measures for IFSs and the extended measures for IvIFSs are based on the axiom, which states that entropy should be maximal and equal to 1 for $\mu_A = \nu_A$. For example, assume that we have a survey of a set X of n electors, who will vote for/against building of a nuclear power plant. Let us assume that each elector x_i belongs to

- a set of electors voting for – to the extent $\mu(x_i)$,
- a set of electors voting against – to the extent $\nu(x_i)$,
- a set of electors abstaining – to the extent $\pi(x_i)$, where $\pi(x_i) = 1 - \mu(x_i) - \nu(x_i)$.

From the viewpoint of decision making, one is interested in discriminating the following cases:

- we have the same number of electors voting for as against, $\mu(x_i) = \nu(x_i)$
- we have no elector voting for/against, $\mu(x_i) = \nu(x_i) = 0$.

In fact, it is easily noted that in IFSs, information entropy measures should be different for $F = \langle x, 0, 0\rangle$ where there is no information at all and for $A = \langle x, 0.4,\ 0.4\rangle$ or $B = \langle x, 0.3,\ 0.3\rangle$ where positive information is as large as negative information. Similarly in IvIFSs, information entropy measures should be different for $F = \langle x, [0, 0], [0, 0]\rangle$ where there is no information at all and for $A^I = \langle x, [0.4, 0.5], [0.4, 0.5]\rangle$ or $B^I = \langle x, [0.2, 0.3], [0.2,\ 0.3]\rangle$ where interval positive information is the same as interval negative information. In the case of data represented in terms of ordinary FSs, information conveyed is expressed by a membership function and its entropy reaches maximum equal to 1 for the most fuzzy set, i.e. $\mu_A = \nu_A = 0.5$. In the case of data represented in terms of IFSs, information is expressed by membership and non-membership functions (“responsible” for the positive and negative information, respectively) and hesitation margin related to a lack of information (epistemic uncertainty) (Dubois & Prade, 2012). In this case, information entropy should measure not only the fuzziness, the gradualness between membership and non-membership degrees, but also the epistemic uncertainty. The equality $\mu_A = \nu_A \ne 0.5$ reflects a fuzzy set with an uncertainty related to a lack of information. The set is most “uncertain” and fuzzy only for $\mu_A = \nu_A = 0$ (zero information). Similarly, in the case of IvIFSs, information entropy measures the gradualness between interval membership and interval non-membership degrees, the epistemic uncertainties connected with their interval degrees and with the interval hesitancy margin. The IvIFS is most “uncertain” and fuzzy only for $M_{A^I} =\ N_{A^I} = 0$. The properly constructed information entropy measure should differentiate and correctly evaluate these cases due to amount of knowledge useful from the viewpoint of decision making.