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A multi-machine multi-product EPQ problem for an imperfect manufacturing system considering utilization and allocation decisions

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ABSTRACT

This paper considers a multi-product problem with non-identical machines. This manufacturing system consists of various machine types with different production capacities, production costs, setup times, production rates and failure rates. One of the major issues in the planning phase of a manufacturing system is to take the best decision about which machines must be utilized to manufacture which items. As a result, the decision makers face three critical questions: what machines must be purchased, which items should be allocated to each machine, and what is the optimal cycle length. These decisions must be made to minimize system costs including utilization, setup, production, holding and scrap costs. The multi-machine multi-product economic production quantity (EPQ) problem for an imperfect manufacturing system is formulated as a mixed integer non-linear programming (MINLP), where the convexity property of multi-product single machine EPQ model is used to convert the problem into a bi-level decision-making problem. In the first level, decisions about machine utilization and items allocation are made. After, in the second level the optimal cycle length for each machine is determined. To solve the problem at hand, a hybrid genetic algorithm (HGA) is proposed integrating genetic algorithm and derivatives method. In the proposed HGA, the solutions of the first level are obtained randomly and then, for the second level, the derivatives method is applied to obtain optimal cycle length based on solutions of the first level. Finally, the results of HGA method are compared to the results of general algebraic modeling system (GAMS) and it is found that HGA method has better and more efficient results. Also, a numerical experimentation and a sensitivity analysis of the model are done.

1. Introduction

Definitely, the first inventory model dates back to the early twentieth century, when Ford Whitman Harris in 1913 presented the well-known economic order quantity (EOQ) inventory model in United States of America (Harris, 1913). In this classical inventory model, the optimal order quantity is calculated considering only holding and ordering costs. Harris's (1913) inventory model provides the optimal order quantity for factory managers, instead of previous values obtained experimentally by trial and error or by the managers' intuition. It is important to mention that this inventory model is also known as Wilson formula; because Wilson was a consultant in US factories that recommended using the EOQ formula to many managers, and he played a significant role in promoting the Harris (1913) inventory model. A biography of Harris and a review of the early life of this formula are given in Erlenkotter (1990).

Additional information related to the classification of EOQ and EPQ inventory models up to 2013 is given in Glock, Grosse, and Ries (2014). On the other hand, 219 papers are explored by Andriolo, Battini, Grubbström, Persona, and Sgarbossa (2014) to create an insight into the evolution of EOQ inventory model in a century. Moreover, a special issue in honor to Ford Whitman Harris with 41 papers related to EOQ and EPQ inventory models was published in 2014; see Cárdenas-Barrón, Chung, and Treviño-Garza (2014).

The advancement of technology and the construction of new factories around the world, encouraged production managers to update their inventory system taking into account both production and demand quantity. As a result, there was a need to develop inventory models which considers production capacity. In this direction, Taft (1918) modified the EOQ model considering production capacity and proposed the economic production quantity (EPQ) inventory model which led to holding and manufacturing costs reduction.

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Wide application and popularity of these two inventory models is a result of their simplicity of concept and practicality. Nevertheless, some restrictive assumptions and conditions, limit their applicability in real-world situations. There are a wide range of studies that aim to relax some of these assumptions. For an example of the early efforts of generalizing EOQ model, the dynamic version of this model proposed by Wagner and Whitin (1958) can be mentioned.

The consideration of defective items and breakdowns is an extension of EPQ inventory model. In this direction, Chiu, Wang, and Chiu (2007) optimized the run time for this model where a fraction of defective items is repaired and the rest are scrap items. Moreover, stochastic breakdowns are considered and using some theorems for conditional convexity of model they obtain the optimal boundaries for optimal run time. Following this paper, Chiu and Ting (2010) enhanced the quality of solution by proving the convexity of their solution procedure.

There is a vast literature on inventory models for multiple products on one machine. Maybe Eilon (1957) and Rogers (1958) were the first academicians that investigated the multi-products single machine system. After that, this problem was addressed widely in the papers of Bomberger (1966), Hodgson (1970), Madigan (1968), Stankard and Gupta (1969) and Baker (1970), just to mention a few pioneer papers that deal with multi-products single machine system. Afterwards, Cooke, Rohleder, and Silver (2004), Davis (1990), Fransoo, Sridharan, and Bertrand (1995), Hishamuddin, Sarker, and Essam (2012), Sarker and Newton (2002) followed investigating the multi-products single machine system. Recently, Taleizadeh, Wee, and Sadjadi (2010) developed an inventory model that studies multi-products single machine manufacturing system considering stochastic scrapped production rate, partial backordering and service level constraint. This inventory model obtains for each product the optimal production quantity, the allowable shortage level, and the period length. Simultaneously, Taleizadeh, Najafi, and Niaki (2010) derived an EPQ inventory model with backorders to calculate the optimal lot size and backorders level for multiproduct fabricated in a one machine. Likewise, Taleizadeh, Niaki, and Najafi (2010) built an EPQ inventory model with random defective items, service level constraints and repair failure. Essentially, this inventory model computes the optimal cycle length, optimal lot size and optimal backordered level. After, Taleizadeh, Sadjadi, and Niaki (2011) proposed a multiproduct single machine problem with and without rework allowing backorders. At the same time, Taleizadeh, Shavandi, and Haji (2011) dealt with the multi-product, multi-constraint, single period problem taking into account uncertain demands and an incremental discount situation. Subsequently, Taleizadeh, Cárdenas-Barrón, Biabani, and Nikousokhan (2012) derived an EPQ inventory model with rework process for multi-products in one machine and obtained the optimal cycle length as well as the optimal production quantity for each product. At the same time, Ramezanian and Saidi-Mehrabad (2012) developed a mixed integer nonlinear programming (MINLP) model to solve a multi-product unrelated parallel machines scheduling problem taking into account that the manufacturing system can generate imperfect products. Later, Taleizadeh, Wee, and Jalali-Naini (2013) derived an EPQ inventory model with random defective items and failure in repair for multiproduct in one machine system. Afterward, Taleizadeh, Jalali-Naini, Wee, and Kuo (2013) optimized a joint total cost for an imperfect, multi-product production system with rework subject to budget and service level constraints. In a subsequent paper, Taleizadeh, Cárdenas-Barrón, and Mohammadi (2014) addressed and examined an EPQ inventory model with interruption in the process, scrap and rework. Their inventory model considers multiple products and all products are processed in a one machine. Pasandideh, Niaki, Nobil, and Cárdenas-Barrón (2015) developed an EPQ model for a multi-product single-machine lot-sizing problem with defective items including rework and scrap, where reworks are classified into several groups based on failure severity.

This research work develops a multi-product single-machine EPQ inventory model for a multi-machine lot-sizing problem with scrapped items. The proposed inventory model considers inventory decisions, allocations and machine utilization all together; the decision must be made about whether utilizing each machine or not, where the production system includes several machines.

The aim is to determine the optimal cycle length of each product, the number and type of machines to be purchased and allocation of products to machines such that the total cost, including utilization, setup, holding, production and disposal costs is minimized. Besides, the available total budget and the available total production floor space are limited.

Moreover, real world requirements in the design phase of production system, i.e. type and number of machines that have to be purchased, are considered. The reason is due to the fact that there exist different price, capacity, production rate, failure rate and production cost for each machine. The multi-machine multi-product EPQ problem for an imperfect production system is formulated as a mixed integer non-linear programming (MINLP) and it is transformed into a bi-level decision making problem. To solve this problem, a hybrid genetic algorithm (HGA) that uses the bi-level decision making process is proposed. It is important to remark that the HGA obtains efficient solutions to this problem.

The organization of the remainder of the paper is as follows. Section 2 presents the assumptions and defines the multi-machine multi-product EPQ problem for an imperfect production system. Section 3 provides the notation and develops the mathematical formulation of the multi-machine multi-product EPQ problem for an imperfect production system. Section 4 develops the hybrid genetic algorithm (HGA) to solve the problem at hand. Section 5 presents a numerical experimentation and a sensitivity analysis. Finally, Section 6 gives a conclusion and some future research directions.

2. Problem definition and assumptions

In real world problems, there are different options to purchase machines, considering different factors like: production rate, floor space limitation for production, budget constraint, and so forth. As a result, production managers cope with decisions about minimizing machine utilization expenditure and inventory costs simultaneously. Moreover, when factory produces more than one item, factory may need to buy more than one machine. This study investigates this problem.

Consider a multi-machine production system producing m different items. The problem is an EPQ problem with unrelated parallel-machine in which utilized production machines are considered. It is an extension of the single-machine multi-product EPQ problem with defective items, in which the determination of machine's number and items allocation is considered simultaneously. Each machine has a particular failure rate based on the characteristics of product. The proposed inventory model minimizes total cost of the inventory system, including utilization, setup, production, holding and disposal costs. Furthermore, it finds the answer to the following questions:

– What machines are optimal to be utilized?
– What is the optimal allocation of items and machines?
– What is the optimal length cycle of each machine?

Knowing the length cycles of each machine, one can calculate the optimal production of each item on it. The following assumptions are considered in this study:

- Shortages are not allowed.