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Double auction mechanisms on Markovian networks ☆

Xiaojing Xu [a], Jinpeng Ma [b,c,*], Xiaoping Xie [a]

[a] School of Mathematics, Sichuan University, Chengdu 610064, PR China
[b] Department of Economics, Rutgers University, Camden, NJ, USA
[c] School of Economics, Sichuan University, Chengdu 610064, PR China

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ABSTRACT

This paper studies the double auction (DA) mechanism in Ma and Li (2011) for a class of exchange economies. We extend their results to more general cases where sellers and buyers each form a complex time non-homogeneous Markovian chain, as specified in Ram et al. (2009), in the communication of their private information. A numerical example is also provided. Both bubbles and crashes are observed in the example, consistent with results of our theorems. Our example and theoretical results provide new evidence that a DA mechanism, widely utilized in real exchange markets, may contribute to the excess volatility identified in Shiller (1981) and LeRoy and Porter (1981).

1. Introduction

Agents in an economy are often connected with one another through a network. Communication of private information in a network is likely local in the sense that agents may just pass their private information to their close neighbors (or friends), those with whom the agents have a direct link (see e.g. Ellison (1993)). The network structure does not always remain constant. Oftentimes it may evolve. Indeed, a person's connections with their neighbors may drop off for one reason or be added for another. Such a complex network structure can be best modeled with a time non-homogeneous Markovian chain with economic agents seen as the states, in a style specified in Ram, Nedić, and Veeravalli (2009).

This paper addresses the issue of efficiency of a dynamic DA mechanism for a class of exchange economies when buyers and sellers each form such a Markovian network. That is, we want to know if the DA mechanism can generate a price sequence that converges to a Walrasian equilibrium of the underlying economy when agents form a time-varying Markovian network. Such an issue regarding the DA mechanisms has been addressed in Ma and Li (2011) when agents are connected in a two ring structure (Fig. 2) or agents are fully connected and pass their private information to others (including themselves) with equal probability for every individual.

Next we will introduce our DA mechanisms non-technically and make a case for how they are related to a DA mechanism used in a real exchange market. We will motivate the study in this paper and explain the limitations of theoretical DA mechanisms in modeling a real exchange market in Section 1.2. The details of a time non-homogeneous Markovian chain will be introduced in Section 2.

1.1. DA mechanisms and real exchange markets

Stocks, among many other assets, are traded under a DA mechanism in a real exchange market, where an exchange of shares must be done between one buyer and one seller, one order at a time. Thus the price of a stock at each moment in time is solely determined by a weighted average of the bid and ask prices of an executed trade. The following table lists three typical examples of posted information available publicly by the market close on Nov. 15, 2013. Similar information is also provided by a brokerage house at each moment during a trading day.

In Table 1, the first number in the row "Bid" ("Ask") of a stock is the bid (ask) price and the second one is the bid (ask) size or number of shares to buy (sell) for that stock. Because trades only occur when the bid is higher than the ask, the bid and the ask in Table 1 are just the best bid and ask offers that have not been executed by market close. Now take Google as an example. We may consider three cases when the market reopens the next day (assume the bid and the ask at the close both stay put when the market reopens the next day): (a) there is a market order to buy 600 shares. Then the next executed best price must be $1033.80 and 600 shares will

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* Corresponding author at: Department of Economics, Rutgers University, Camden, NJ, USA. Tel.: +1 610 4088560.

E-mail addresses: xuxiaojing0603@126.com (X. Xu), jinpeng@crab.rutgers.edu, jinpeng_m@yahoo.com (J. Ma), xpxiec@gmail.com (X. Xie).

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Table 1
Three examples shown after market close.

Name	GOOGLE	APPLE	METLIFE
Ticker	GOOG	AAPL	MET
Posted price	1033.56	524.991	52.01
Bid	1032.05 × 200	524.96 × 200	51.87 × 100
Ask	1033.80 × 600	525.01 × 100	52.00 × 100

4 pm, November 15, 2013. Data Source: Etrade.

be transferred from the seller (s), who is (are) holding the current best ask offer, to the buyer; (b) there is a market order to sell 200 shares. Then the next executed best price must be \$1032.05 and 200 shares will be transferred from the seller to the buyer (s), who is (are) holding the current best bid offer; (c) there is a pair of new bid and ask (bid, ask) prices = (1033.60 × 100, 1033.50 × 100). Such a pair of orders will be the next best offers, ahead of the pair of posted bid and ask in Table 1. The two orders will be executed, say, at price \$1033.55, since the best bid price is higher than the best ask price. 100 shares will be sold from the seller to the buyer.

A DA mechanism in practice is dynamic in nature. Thus, we consider the price evolution as a dynamic process $x_k, k = 0, 1, 2, \ldots$, starting with some given $x_0 > 0$. For the example above, let $x_k = \$1033.56$. Then $x_{k+1} = \$1033.80$ in case (a), \$1032.05 in case (b) and \$1033.55 in case (c). The question is how to relate x_{k+1} to x_k in the three cases in one framework.

Let ψ_{k+1} be the best ask price and φ_{k+1} be the best bid price such that $\varphi_{k+1} \geqslant \psi_{k+1}$. Then there is a trade and the executed price is given by

$$x_{k+1} = \alpha\psi_{k+1} + (1-\alpha)\varphi_{k+1}, \quad \alpha \in [0, 1],$$

where α is some weight. Such a DA mechanism is also called an α–DA mechanism in Chatterjee and Samuelson (1983), Wilson (1985), where the analysis of an α-DA mechanism for selling one share is provided using a strategic form game with incomplete information. We are interested in the convergence of the sequence $\{x_k\}$ that is generated by a dynamic DA mechanism for an economy that allows us to sell a finite number of shares of a finite number of stocks. To address this question, we define (in the example of selling a finite number of shares of a single stock)

$$a_k = -\frac{\psi_{k+1} - x_k}{\text{ask size}}, \quad b_k = \frac{\varphi_{k+1} - x_k}{\text{bid size}}.$$

The ask (bid) size can be seen as the quantity of shares supplied (demanded) at price x_k from the seller (buyer) who places the best ask (bid) price, ψ_{k+1} (φ_{k+1}). Notice that the posted price x_k is public information available before agents submit their buy or sell order, which is either a market or limited order. Here a_k and b_k are called the bid and ask stepsizes at k, respectively. For the Google example, in case (a), $b_k = 0.0004$ and $\alpha = 0$. In case (b), $a_k = 0.00755$ and $\alpha = 1$. In case (c), $a_k = 0.0006, b_k = 0.0004$ and $\alpha = \frac{1}{2}$. Thus, for a sequence of executed prices $x_k, k = 0, 1, 2, \ldots$, there will be two derived sequences of stepsizes a_k and $b_k, k = 0, 1, 2, \ldots$.

In the above, the two sequences $\{a_k\}$ and $\{b_k\}$ are obtained from the bid $\{\varphi_{k+1}\}$ and the ask $\{\psi_{k+1}\}$ sequences. But we can also derive the bid and ask sequences from the two given sequences of stepsizes $\{a_k\}$ and $\{b_k\}$ as follows:

$$\psi_{k+1} = x_k - a_k \cdot \text{ask size}, \quad \varphi_{k+1} = x_k + b_k \cdot \text{bid size}.$$

Such a structure has a major advantage because it allows us to study an economy for selling multiple stocks when x_k is seen as a price vector. To study the convergence of $\{x_k\}$, we have to have restrictions on $\{a_k\}$ and $\{b_k\}$ (equivalently on $\{\psi_{k+1}\}$ and $\{\varphi_{k+1}\}$). Moreover, we need to be more specific about the environment in which a DA mechanism operates. In this paper we study a class of economies represented by the following general form Bertsekas (2010):

$$\mathcal{P} \text{ minimize } F(y) \equiv f(y) + g(y)$$
$$\text{subject to } y \in Y,$$

where

$$f = \sum_{i=1}^{m} f_i \quad \text{and} \quad g = \sum_{j=1}^{n} g_j.$$

For all $i \in I = \{1, 2, \ldots, m\}$ and $j \in J = \{1, 2, \ldots, n\}, f_i : Y \to R$ and $g_j : Y \to R$ are real-valued (possibly non-differentiable) convex functions and Y is a nonempty convex set of finite dimensions. Let $F^* = \inf_{y \in Y} F(y)$ and $Y^* = \{y \in Y | F(y) = F^*\}$.

We may consider all i agents the sellers and all j agents the buyers. Ma and Li (2011) demonstrated that a large class of quasi-linear economies with indivisible (as well as divisible) goods or assets can be modeled with $\mathcal{P}$. The noted assignment problem and the two-sided job matching model (Kelso & Crawford, 1982) are two typical examples that can be modeled by $\mathcal{P}$. More important, Y^* is also the set of Walrasian equilibrium prices of an underlying economy (Ma & Nie, 2003). Thus, to address the efficiency of a DA mechanism, we ask the following question: under what conditions on the two stepsizes $\{a_k\}$ and $\{b_k\}$ does a price sequence generated by a DA mechanism, $x_k, k = 0, 1, 2, \ldots$, converge to an equilibrium in Y^*? The diminishing stepsize rule (Nedić & Bertsekas, 2001; Ram et al., 2009) is a natural choice.[1] But Ma and Li (2011) found that the diminishing stepsize rule alone is not sufficient for the convergence of the sequence $x_k, k = 0, 1, 2, \ldots$, to an equilibrium. It turns out that the following inequality[2] is critical:

$$\sum_{k=0}^{\infty} |b_k - \lambda a_k| < +\infty \tag{1.1}$$

for some positive constant λ. That is, beyond conditions on $\{a_k\}$ and $\{b_k\}$, certain coordination between the two sequences $\{a_k\}$ and $\{b_k\}$ must be present as well for the price sequence $\{x_k\}$ to converge to an equilibrium in Y^*. For a technical reason, this condition is needed in order to use the supermartingale convergence theorem (Lemma 1 in Bertsekas & Tsitsiklis (2000)) in the proofs of Theorems 4.1 and 4.2 (see the remark after Theorem 4.1 for detail).

1.2. Literature

As alluded above, for a large class of quasi-linear economies, Y^* can be shown to be the set of Walrasian equilibrium prices (Ma & Li, 2011; Ma & Nie, 2003). For an agent i or j, the set of subgradients $-\partial g_j(y)$ or $\partial f_i(y), y \in Y$, is the closed convex-hull of the demand or supply, using the Fenchel duality (Ma & Nie, 2003). Thus, a subgradient $-\bigtriangledown g_j(y)$ in $-\partial g_j(y)$ is j's bid size at prices y and a subgradient $\bigtriangledown f_i(y)$ in $\partial f_i(y)$ is i's ask size. For example, the noted Walrasian price adjustment process uses the information of total demand and supply. Such a process can be modeled with the subgradient method (via the Fenchel duality).

Subgradient Method (Bertsekas & Tsitsiklis, 2000): Assume $m = n$. For $k = 0, 1, 2, \ldots$, let

$$x_{k+1} = P_Y(x_k - a_k \bigtriangledown (f+g)(x_k)),$$

where $x_0 \in Y$ and P_Y is the Euclidean projection onto the set Y. a_k is the stepsize rule of the price adjustment process $x_k, k = 0, 1, 2, \ldots$.

[1] Diminishing stepsize rule: *The two sequences* $\{a_k\}$ *and* $\{b_k\}$ *of stepsizes are such that* (i). $a_k > 0$ *and* $b_k > 0$; (ii). $\sum_{k=0}^{\infty} a_k = +\infty$ *and* $\sum_{k=0}^{\infty} b_k = +\infty$; (iii). $\sum_{k=0}^{\infty} a_k^2 < +\infty$ *and* $\sum_{k=0}^{\infty} b_k^2 < +\infty$.

[2] For the case where $m = n$, the condition will be given in (4.10) and (4.21) when $m \neq n$.