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Adaptive image sampling through establishing 3D geometrical model

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ABSTRACT

Adaptive sampling for high dimensional manifold attracts much attention from related fields. The principal curvature based strategy is one of the popular methods. However, principal curvature estimation remains an open problem. Considering the relationship between geodesics and the principal curvatures of manifold, we transform the optimized sampling density computation into the problem of uniform sampling in the geodesic metric of manifold. Therefore, two well studied uniform sampling methods such as Poisson disk and farthest point strategy are used. For image sampling, a 3D geometrical metric model is built based on mean shift. Mean shift value is applied to describe the image grey information and taken as the height of this model. Uniform sampling is implemented to generate samples with blue noise properties on the 3D model surface. Then, adaptive results are obtained when these samples are projected back to the original 2D image. In contrast to previous methods, this strategy is flexible and can be easily extended to unorganized points simplification or mesh coarsening. Extensive experiments demonstrated the effectiveness of the proposed method.

1. Introduction

Signal sampling is widely applied in data transmission, image compression, mesh coarsening and etc. (Correal, Pajares, & Ruz, 2014; ElAlami, 2011; Wei & Rui, 2011). In the field of image processing, the traditional theory is that the uniformly and irregularly distributed samples (blue noise properties) are best for image reconstruction (Hiller & Keller, 2001; Ostromoukhov & Jodoin, 2004). Irregularity is to avoid aliasing effect caused by regular subsampling. Uniformity means that the densities of samples are closely constant, thus providing an equal amount of information about each region of an image. Two classic methods to obtain the blue noise properties distribution are farthest point strategy and Poisson disk sampling. Eldar proved that the optimized sampling strategy for the 1D case should pick the point at the middle of the longest unsampled line segment. The extension of the 1D optimal rule to 2D case in a geometrical way results in the classic farthest point strategy (FPS) method (Eldar, Lindenbaum, Porat, & Zeevi, 1997). Poisson disk sampling sets an exclusive region for each sample to ensure that the distance between any two samples is larger than a threshold (Cook, 1986). To achieve further high quality blue noise distribution, the relaxation scheme iterates to relocate the original samples (Dunbar & Humphreys, 2006; Lagae & Dutre, 2008). The similar works can also be found in Cohen, Shade, Hiller, and Deussen (2003) and Lagae and Dutre (2006).

Adaptive sampling is in great demands in most situations. It usually means that more points should be sampled in the parts of image rich in detail and fewer samples are chosen in smooth subregions. One of the advanced strategies is the adaptive farthest point sampling (Eldar et al., 1997), in which a local weight function is defined for the generated voronoi vertexes rather than taking account of the local image information when generating voronoi vertexes. A method, preferred by many applications for its high speed and simplicity, is to locate samples according to the local skewness of image illumination (Ramponi & Carrato, 2001). Samples will be concentrated in the areas with great skewness, where image edges or sharp features exist. Other attempts include the deformation or relaxation methods (Wei & Rui, 2011; Ostromoukhov & Jodoin, 2004), the spectral analysis (Oeztireli, Alexa, & Gross, 2010) and adaptive mask (Devir & Lindenbaum, 2012). The common characteristic of these methods is that they try to perform adaptive sampling on basis of the uniform and irregular samples with blue noise properties. Their defects are that the samples may be rather uneven and their computations will be complex when the global optimization is applied.

From another viewpoint, the theoretical exploration about high dimensional manifold adaptive sampling remains the hot topic in related fields of signal processing. Based on the one-dimensional signal sampling theorem of Shannon (Marks, 1991), Apleboim and Saucan demonstrated that the optimized sampling ratio for

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higher dimensional manifold should be proportional to the local maximal principal curvature (Appleboim, Saucan, & Zeevi, 2007; Saucan, Appleboim, & Zeevi, 2008). According to this theorem, adaptive image sampling is achieved as follows. Firstly, the image was divided into many unoverlapped blocks and the maximal principal curvature of each block was assessed using finite element techniques (Appleboim et al., 2007). Then, they obtained sparse samples relative to the original one in each block, whilst maintaining good reproduction as a whole. However, the samples' densities are obviously discontinuous among the adjacent blocks, which will be disadvantageous for subsequent image processing. Meanwhile, they pointed out that the principal curvature estimation is an open problem and it would be beneficial if the principal curvature could be replaced by or approximated to some extent by more accessible intrinsic curvature measures such as the sectional/Ricci/scalar curvatures (Appleboim et al., 2007). To address these problems, this paper intends to transform the curvature based sampling strategy into the problem of uniform sampling in the geodesic metric of the manifold by establishing the relationship between geodesics and curvature.

The rest of this paper is organized as follows. Section 2 presents our sampling strategy. Section 3 describes the method of building three dimensional geometrical metric model. The weighted geodesic distance computation method and two uniform sampling methods are described in Sections 4 and 5, respectively. In Section 6, the detailed implementation issue is presented. Some experiments are conducted and results are discussed in Section 7. Finally, conclusion and possible future research are provided.

2. Our methodology

The sampling theorem in Appleboim et al. (2007) is as follows.

Theorem 1. *Let $\sum^n, n > 2$ be a connect, not necessarily compact, smooth manifold, with finitely many compact boundary components. Then there exists a sampling scheme of $\sum^n$, with a proper density $D = D(k(p))$, where $k(p) = max(|k_1|, \ldots, |k_n|)$, and $k_1, \ldots, k_n$ are the principal (normal) curvatures of $\sum^n$, at the point $p \in \sum^n$.*

To address the difficulty in estimating the principal curvature, we resort to the geodesics. As a typical problem in computer graphics, geodesics is defined to be the shortest path connecting two points on the manifold surface (Seong, Jeong, & Cohen, 2009). The geodesics from point A to B on a sphere will be the shorter arc of the great circle passing through A and B. The length of this arc is their geodesic distance. For all arcs connecting A and B on the sphere surface, the curvature of the geodesic arc is among the smallest. Moreover, the geodesic distances between any two points on sphere are all equal only if the Euclidean distances between two points are the same. This is simply due to the fact that a sphere is isotropic, i.e., the principal curvatures of all points on a sphere are equal to *1*/*r*, where *r* is the radius of sphere. In the case of the anisotropic surface, however, geodesic distances between two points are diverse when the two points are located in different subregions. Roughly, the more rugged the subregion is, the longer the geodesic distance is. Meanwhile, it is obvious that the principal curvature is proportional to the rough degree. Then, the principal curvature is highly related to the geodesics (Seong et al., 2009; Wen, Jiang, & Wen, 2008). I.e., the principal curvature can be estimated by the average geodesic distance from each point to its neighbors in the local area. Based on this fact and Theorem 1, the following theorem can be deduced.

Theorem 2. *Let $\sum^n, n > 2$ be a connect, not necessarily compact, smooth manifold, with finitely many compact boundary components. Then there exists a sampling scheme of $\sum^n$, with a proper density $D = D(\bar{g}(p))$, where $\bar{g}(p) = \frac{1}{k}\sum_i g_{pi}$, and where g_{pi}, $i = 1, \ldots k$ is the geodesic distance from the point $p \in \sum^n$ to the point $i \in \sum^n$, i is the one of neighbors of p. k is the number of neighbors.*

Theorem 2 means that the average geodesic distance from each point to its neighbors can be adopted to estimate the sampling ratio for the related subregion. Then, a natural and direct solution to higher manifold sampling is to perform uniform sampling by taking manifold surface distance or geodesic distance as basis (in general uniform sampling where Euclidean distance is taken to represent the points' spatial distance, the sampling ratio for each point on the 2D plane is equal). According to manifold learning, geodesic metric represents a space, in which, the spatial distance between any two points is defined as their corresponding geodesic distance in Euclidean metric. Thereby, the optimized strategy is to perform uniform and irregular sampling (blue noise properties) in geodesic metric. Therefore, our adaptive image sampling method is as follows.

Firstly, a new 3D geometrical metric model is established based on mean shift in Section 3.1. For the established 3D model, the height describes the grey variation between current point and its neighbors. Then, by borrowing the method of geodesic distance computation from computer graphics, two well studied uniform sampling methods are applied on our 3D model using weighted fast marching: Poisson disk and farthest points sampling. Finally, non-uniform sampling results can be obtained when these samples are projected back onto the original image. Illustrated with a 1D case in Fig. 1, the original points (red dotes) are regularly distributed on the horizontal line and the curve's height represents each point's grey difference from its neighbors. Then, we perform random and uniform sampling along the curve to obtain three samples *A*, *B* and *C*. The geodesic distances $\widetilde{AB}$ and $\widetilde{BC}$ are same. Whereas, their densities are adaptive to the local grey variations after being projected onto the original line as A', B' and C'.

3. 3D geometrical metric model

3.1. Mean shift

Mean shift was introduced by Fukunaga in 1975. Let *S* be a finite data set in Euclidean space *X* and *k* be the kernel function of *X*.

$$k = \begin{cases} 1 & \|x\| \leqslant \gamma \\ 0 & \|x\| > \gamma \end{cases} \tag{1}$$

The mean value at $x \in X$ can be described as:

$$m(x) = \sum_{s \in S} k(s - x)s \Big/ \sum_{s \in S} k(s - x) \tag{2}$$

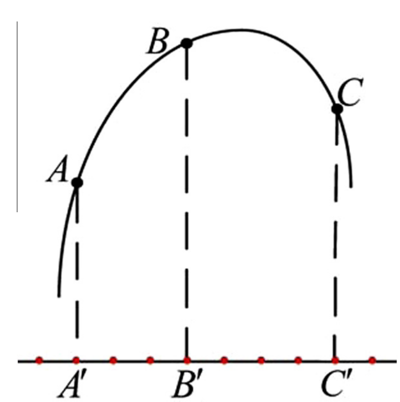

Fig. 1. Principle of the proposed adaptive image sampling.