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A hybrid particle swarm optimization algorithm for satisficing data envelopment analysis under fuzzy chance constraints

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ABSTRACT

This paper presents a new satisficing data envelopment analysis (DEA) model with credibility criterion, in which the inputs and outputs are assumed to be characterized by fuzzy variables with known membership functions. When the inputs and outputs are mutually independent trapezoidal fuzzy variables, we turn the proposed satisficing DEA model into its deterministic equivalent programming problem. For general fuzzy input and output variables, we design a hybrid particle swarm optimization (PSO) algorithm by integrating approximation method, neural network (NN) and PSO algorithm to solve the proposed DEA model, in which the approximation method is used to compute the credibility functions, NN is used to approximate the credibility functions, and PSO is used to find the optimal solution of the proposed DEA problem. Furthermore, the sensitivity analysis of the proposed model is discussed. Finally, we perform a number of numerical experiments to demonstrate the effectiveness of the hybrid PSO algorithm. The computational results show that the designed hybrid PSO algorithm outperforms the hybrid genetic algorithm (GA) in terms of runtime and solution quality.

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1. Introduction

Data envelopment analysis (DEA) was initiated and developed by Charnes, Cooper, and Rhodes (1978), it is a non-parametric technique for evaluating the relative efficiency of a set of homogenous decision making units (DMUs). Each of these DMUs is responsible for converting multiple inputs into multiple outputs. Since then, there has been a large number of research on DEA models in the literature, such as BCC model (Banker, Charnes, & Cooper, 1984), FDH model (Petersen, 1990) and SBM model (Tone, 2001). Recently, Mecit and Alp (2013) developed the restricted DEA model to take into account the relations between inputs/outputs variables using correlation coefficients; Washio and Yamada (2013) formulated the RBM model to evaluate DMU from a different standpoint. Other well-known DEA models can be found in Cooper, Seiford, and Tone (2007) and Cook and Seiford (2009). Owing to these important works, DEA has been extensively used for evaluating the performance of many activities, such as education systems, health care units, productivity evaluation on research institutions, and firm's financial statement analysis.

Traditionally, the coefficients of DEA models, i.e. the data of inputs and outputs of different DMUs, are assumed to be measured with precision. However, in many real-world problems, the observed values of the input and output data are sometimes imprecise or uncertain due to the measurement errors or data entry errors. Under this consideration, some authors incorporated stochastic input and output variations into the DEA models. For instance, Sengupta (1982) generalized the CCR model by defining the measure of efficiency of a DMU as the maximum of the sum of the expected ratio of weighted outputs to weighted inputs and a reliability function subject to several chance constraints; Land, Lovell, and Thore (1993) and Olesen and Petersen (1995) developed two different chance-constrained DEA models, and Cooper, Deng, Huang, and Li (2004) covered the topic of congestion as treated in the stochastic DEA.

On the other hand, some researchers have proposed various fuzzy approaches to deal with the fuzziness in DEA models. For example, Guo (2009) formulated fuzzy DEA models for evaluating the efficiencies of objects with fuzzy input and output data; Wen and Li (2009) established a fuzzy DEA model based on the credibility measure; Azadeh, Moghaddam, Asadzadeh, and Negahban (2011) provided an integrated fuzzy simulation-fuzzy DEA algorithm to cope with a special case of single-row facility layout problem; Wang and Chin (2011) used fuzzy expected value approach to formulate fuzzy DEA model; Angiz, Emrouznejad, and Mustafa (2012) introduced the concept of "local α-level" to develop a multi-objective linear programming to measure the efficiency of DMUs under uncertainty; Muren, Ma, and Cui (2012) built a fuzzy DEA model based on sample decision making unit; Puri and Yadav (2013) developed a new method for ranking the DMUs on the basis of fuzzy input mix-efficiency using α-cut approach and a fuzzy

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correlation coefficient method using expected value approach. In addition, Qin and Liu (2010) presented a new fuzzy random DEA model when randomness and fuzziness coexisted in a decision system; Qin, Liu, and Liu (2011) developed a DEA model with type-2 fuzzy inputs and outputs to deal with linguistic uncertainties as well as numerical uncertainties; Tavana, Shiraz, Hatami-Marbini, Agrell, and Paryab (2012) proposed three fuzzy DEA models with respect to probability-possibility, probability-necessity and probability-credibility constraints. The other published researches and bibliographies have appeared in Hatami-Marbini, Emrouznejad, and Tavana, 2011; Liu, Lu, Lu, and Lin (2013).

However, to the best of our knowledge, the fuzzy approaches mentioned above suffer from a major drawback: they do not incorporate the idea of "satisficing" into fuzzy DEA models. The "satisficing" concept has the origin of psychology as an alternative to the assumption of "optimizing" behavior, which is extensively used in economics. Inspired by these facts, this paper will construct a novel satisficing DEA model with credibility criterion under fuzzy environment. Then we conduct the sensitivity analysis of the proposed model. In order to solve the satisficing DEA model, we design a hybrid PSO algorithm by incorporating the approximation method, NN and PSO algorithm.

The rest of this paper is organized as follows. In Section 2, we first formulate a new satisficing DEA model with fuzzy inputs and outputs. Then the concept of satisficing-efficiency is defined. After that, we convert the proposed DEA model into its deterministic equivalent form when the fuzzy inputs and outputs are mutually independent trapezoidal fuzzy variables. Section 3 discusses the issue of sensitivity analysis of the proposed model. In Section 4, we first suggest an approximation approach to compute the objective and the chance functions in the satisficing DEA model. Then, we design a hybrid PSO algorithm by integrating approximation method, NN and PSO algorithm to solve the proposed model. The intent of Section 5 is to perform numerical experiments to illustrate the feasibility and effectiveness of the hybrid PSO algorithm, and analyze the DMUs' relative efficiency. We also compare the designed hybrid PSO algorithm with the hybrid GA in this section. Finally, Section 6 concludes this paper.

2. Formulation of satisficing DEA model

DEA is a method for evaluating the relative efficiency of DMUs with precise inputs and outputs. However, due to many reasons like the data entry error and the noise of the data, we cannot obtain the perfect information about each DMU in real applications. In the current development, we attempt to construct a new satisficing DEA model with fuzzy inputs and outputs, which are characterized by fuzzy variables with known membership functions. The notations required to build our DEA model are collected in Table 1.

Table 1
List of notations in model (1).

Notations	Definitions
$\tilde{x}_{i0}$	The ith fuzzy input of DMU$_0$, i = 1,2, ...,m
$\tilde{x}_{ik}$	The ith fuzzy input of DMU$_k$, i = 1,2, ...,m, k = 1,2,...,n
$\tilde{y}_{j0}$	The jth fuzzy output of DMU$_0$, j = 1, 2,...,r
$\tilde{y}_{jk}$	The jth fuzzy output of DMU$_k$, j = 1, 2,...,r, k = 1,2,...,n
u_i	The weight of the ith fuzzy input, i = 1,2,...,m
υ_j	The weight of the jth fuzzy output, j = 1,2,..., r
$\alpha_k \in (0,0.5]$	A scalar specified in advance as an allowable chance of failing to satisfy the constraint with which it is associated, k = 1,2,...,n
$\beta_0 \in [0,1]$	An aspiration level specified as an efficiency rating which is to be attained
$\beta_k \in [0,1]$	A constant imposed by an outside authority or an individual including superior level of management, k = 1,2,...,n

2.1. A new satisficing DEA model

In this section, we present a new method to formulate a satisficing DEA model with credibility criterion. The proposed satisficing DEA model with credibility constraints is constructed as follows:

$$\begin{aligned} &\max_{u_i,\upsilon_j} \quad \theta = \mathrm{Cr}\left\{\frac{\sum_{j=1}^{r}\upsilon_j\tilde{y}_{j0}}{\sum_{i=1}^{m}u_i\tilde{x}_{i0}} \geqslant \beta_0\right\} \\ &\text{subject to} \quad \mathrm{Cr}\left\{\frac{\sum_{j=1}^{r}\upsilon_j\tilde{y}_{jk}}{\sum_{i=1}^{m}u_i\tilde{x}_{ik}} \leqslant \beta_k\right\} \geqslant 1-\alpha_k, \quad k=1,2,\ldots,n \\ &u_i \geqslant 0,\ \upsilon_j \geqslant 0,\ \forall i,j, \end{aligned} \tag{1}$$

where Cr is the credibility measure defined in Liu and Liu (2002). In model (1), the aim of incorporating "satisficing" concept into efficiency analysis is not only to effect contact with theories of behavior in social psychology, but also to extend the potential uses of DEA model to cases where 100% efficiency can be replaced by aspired levels of performance. Therefore, it offers the individual the opportunity to ascertain a satisficing level for each fuzzy input and fuzzy output.

In model (1), it is the credibility of the output-to-input ratio for the evaluated DMU$_0$ that is maximized under a set of credibility constraints. It also includes a constraint $\beta_k = \beta_0$ with a prescribed credibility level for achieving the "aspiration level" that might exceed the maximum possible value. The credibility constraints are fulfilled by choosing $\upsilon_j = 0$ and $u_i > 0$ for all j and i. Therefore, for continuous fuzzy input and output variables, by the self-duality of credibility measure, we have

$$\mathrm{Cr}\left\{\frac{\sum_{j=1}^{r}\upsilon_j^*\tilde{y}_{j0}}{\sum_{i=1}^{m}u_i^*\tilde{x}_{i0}} \leqslant \beta_0\right\} = 1-\theta^* \geqslant 1-\alpha_0.$$

Here, we use θ^* to indicate the optimal value, and u_i^* and υ_j^* to denote the optimal solutions, and α_0 is the risk of failing to satisfy the constraint of DMU$_0$. Hence, θ^* is the credibility of achieving a value of at least β_0 with the choice of weights u_i^* and υ_j^*, and $1-\theta^*$ is the risk of failing to achieve this value. In addition, it is easy to see that $\alpha_0 \geqslant \theta^*$. Therefore, we introduce the definitions of efficiency in model (1) as follows:

Definition 1. Suppose that $\beta_0 = \beta_{k_0} = 1$. DMU$_0$ is said to be fuzzy-efficient if and only if $\theta^* = \alpha_0$; otherwise, if $\theta^* < \alpha_0$, then it is said to be fuzzy-inefficient.

Definition 2. Suppose that $\beta_0 = \beta_{k_0} < 1$. DMU$_0$ is said to be satisficing-efficient if and only if $\theta^* = \alpha_0$; otherwise, if $\theta^* < \alpha_0$, then it is said to be satisficing-inefficient.

2.2. Deterministic equivalent of satisficing DEA model

In this section, we discuss the equivalent model of the proposed satisficing DEA model when the membership functions of fuzzy inputs and outputs have some special characters. When the fuzzy inputs and outputs are mutually independent trapezoidal fuzzy variables (Liu & Gao, 2007), we have the following properties about credibility constraints:

Theorem 1. *Let $\xi_1, \xi_2, \ldots, \xi_n$, $\eta_1, \eta_2, \ldots, \eta_m$ be mutually independent positive trapezoidal fuzzy variables with $\xi_i = (r_1^i, r_2^i, r_3^i, r_4^i), \eta_j = \left(s_1^j, s_2^j, s_3^j, s_4^j\right)$, $k_i \geqslant 0$, but at least one $k_i > 0$ for i = 1,2,...,n, $h_j \geqslant 0$, but at least one $h_j > 0$ for j = 1,2,...,m and $b > 0$. Then we have:*

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