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# Distributed adaptive containment control of networked flexible-joint robots using neural networks ☆

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ABSTRACT

This study presents a distributed adaptive containment control approach for a group of uncertain flexible-joint (FJ) robots with multiple dynamic leaders under a directed communication graph. The leaders are neighbors of only a subset of the followers. The derivatives of the leaders are unknown, namely, the position information of the leaders is only available for implementing the proposed control approach. The local adaptive dynamic surface containment controller for each follower is designed using only neighbors' information to guarantee that all followers converge to the dynamic convex hull spanned by the dynamic leaders. The function approximation technique using neural networks is employed to estimate the model uncertainties of each follower. It is proved that the containment control errors converge to an adjustable neighborhood of the origin regardless of model uncertainties and the lack of shared communication information. Simulation results for FJ manipulators are provided to illustrate the effectiveness of the proposed adaptive containment control scheme.



## 1. Introduction

The distributed coordination tracking problem for multi-agent systems in the presence of a single leader or multiple leaders has been sustained much interest due to practical applications in various areas. Especially, networked Lagrangian systems, which can describe several mechanical systems such as autonomous vehicles, robot manipulators, and biped robots, have been considered as a main control target in the last few years because they cannot be controlled by the previous distributed control methods (Cao, Stuart, Ren, & Meng, 2011; Ferrari-Trecate, Buffa, & Gati, 2006; Ji, Ferrari-Trecate, Egerstedt, & Buffa, 2008; Lou & Hong, 2010; Li, Ren, & Xu, 2012; Ren, 2007, 2008, 2008; Spanos, Olfati-Saber, & Murray, 2005; Su, Wang, & Lin, 2009; Shi & Hong, 2010) for linear systems with single-integrator or double-integrator dynamics due to their nonlinear property. In the presence of the single leader, distributed tracking control approaches were presented for networked Lagrangian systems (Chen & Lewis, 2011; Chung & Slotine, 2009; Hou, Cheng, & Tan, 2009; Mei, Ren, & Ma, 2011; Min, Sun, Wang, & Li, 2011). However, there have been few research results available on the distributed tracking problem for networked Lagrangian systems in the presence of multiple leaders. A distributed containment control method for Lagrangian systems under a directed graph is recently proposed in Mei, Ren, and Ma (2012) where the control objective is to drive all followers into the convex hull spanned by the multiple leaders. Despite the progress, the aforementioned results for networked Lagrangian systems are not available for networked Lagrangian systems with flexible joints (i.e., networked flexible-joint (FJ) robots with nonlinearities and uncertainties unmatched in the torque input).

On the other hand, the tracking control of a single FJ robot has attracted the attention of many researchers due to the fact that the joint flexibility is an important factor to achieve better link-position tracking performance (Chang & Yan, 2011; Kwan & Lewis, 2000; Tomei, 1991). The backstepping technique (Krstic, Kanellakopoulos, & Kokotovic, 1995) and the dynamic surface design technique (Swaroop, Hedrick, Yip, & Gerdes, 2000), in particular, has been employed for controlling of FJ robots (Abouelsoud, 1998; Bridges, Dawson, & Abdallah, 1995; Nicosia & Tomei, 1991; Yoo, Park, & Choi, 2008). However, the results for single FJ robots in the absence of a networked communication graph cannot be directly applied to the distributed containment control problem of multiple FJ robots in the presence of networked communication among the multiple leaders and followers (i.e., the lack of shared information of multiple leaders).

Motivated by these observations, this paper investigates a distributed adaptive containment control problem of networked uncertain FJ robots in the presence of multiple dynamic leaders under a directed graph topology. Only a small fraction of followers can access the position information about the leaders. A

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distributed adaptive dynamic surface containment control system is designed for the followers described by uncertain FJ robots so that all followers converge to the dynamic convex hull spanned by the dynamic leaders regardless of the lack of information of multiple leaders. The function approximation technique using neural networks is employed to estimate model uncertainties of networked multiple FJ robots included in each communication block. The contribution of this paper is threefold: (i) it is the first trial to design a distributed adaptive containment control system for networked Lagrangian systems with flexible joints; (ii) from the distributed dynamic surface containment control design, the simple local controllers can be designed regardless of the order of the followers and the complexity of communication links; and (iii) the derivative terms of the multiple dynamic leaders are not required to implement the controllers. Adaptive laws for weights of neural networks, the bound of residual approximation error terms, and the bounds of the first derivative terms of the multiple dynamic leaders are derived from the Lyapunov stability theorem. Furthermore, it is shown that all signals of the controlled closed-loop system are semi-globally uniformly ultimately bounded and the containment control errors converge to an adjustable neighborhood of the origin. Finally, simulation results for networked FJ manipulators are provided to illustrate the effectiveness of the proposed approach.

## 2. Preliminaries and problem formulation

### 2.1. Graph theory notions

Consider a group of $N+M$ systems. Then, the communication topology is a directed graph (i.e., digraph for short) $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$ with the set of nodes or vertices $\mathcal{V} \triangleq \{1, \ldots, N+M\}$ and the set of edges or arcs $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. A directed edge $(j,i) \in \mathcal{E}$ means that agent $i$ can obtain information from agent $j$, but not vice versa where $j$ and $i$ are the parent node and child node, respectively. The set of neighbors of a node $i$ is $\mathcal{N}_i = \{j | (j,i) \in \mathcal{E}\}$, which is the set of nodes with edges incoming to node $i$. A directed path from node $i_1$ to node $i_k$ is a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \ldots, (i_{k-1}, i_k)$ in a digraph. A directed tree is a digraph where every node has exactly one parent except for the root and the root has directed paths to every other node. A digraph has a directed spanning tree if there exists at least one agent that has directed paths to all other agents.

### 2.2. Problem statement

Suppose that there exist $N$ followers, labeled as agents 1 to $N$, and $M$ leaders, labeled as agents $N+1$ to $N+M$, in a team. The $N$ followers are represented by the following Euler–Lagrange equations considering flexible joints (Tomei, 1991)

$$\begin{aligned} &M_f(q_f)\ddot{q}_f + C_f(q_f, \dot{q}_f)\dot{q}_f + G_f(q_f) + F_f\dot{q}_f + K_f(q_f - q_{f,m}) + \Upsilon_{f,1}(q_f, \dot{q}_f, q_{f,m}) = 0, \\ &J_f\ddot{q}_{f,m} + B_f\dot{q}_{f,m} + K_f(q_{f,m} - q_f) + \Upsilon_{f,2}(q_f, \dot{q}_f, q_{f,m}, \dot{q}_{f,m}) = u_f, \\ &y_f = q_f, \end{aligned} \tag{1}$$

where $f = 1, \ldots, N, q_f, \dot{q}_f, \ddot{q}_f \in \mathbb{R}^p$ denote the joint position, velocity, and acceleration vectors of the $f$th follower, respectively, $M_f(q_f) \in \mathbb{R}^{p\times p}$is a symmetric positive definite inertia matrix of the $f$th follower, $C_f(q_f, \dot{q}_f) \in \mathbb{R}^{p\times p}$ denotes the Coriolis-centripetal matrix of the $f$th follower, $G_f(q_f) \in \mathbb{R}^p$ is the gravity vector of the $f$th follower, $F_f \in \mathbb{R}^{p\times p}$ is a diagonal, positive definite matrix representing the coefficient of friction at each joint of the $f$th follower, and $q_{f,m}, \dot{q}_{f,m}, \ddot{q}_{f,m} \in \mathbb{R}^p$ denote the actuator position, velocity, and acceleration vectors of the $f$th follower, respectively. The constant positive definite, diagonal matrices $K_f \in \mathbb{R}^{p\times p}$, $J_f \in \mathbb{R}^{p\times p}$, and $B_f \in \mathbb{R}^{p\times p}$ represent the joint flexibility, the actuator inertia, and the natural damping term of the $f$th follower, respectively. The control vector $u_f = [u_{f,1}, \ldots, u_{f,p}]^\top \in \mathbb{R}^p$ is used as the torque input at each actuator of the $f$th follower, and $y_f = [y_{f,1}, \ldots, y_{f,p}]^\top \in \mathbb{R}^p$ is an output vector of the $f$th follower. $\Upsilon_{f,1}(q_f, \dot{q}_f, q_{f,m}) \in \mathbb{R}^p$ and $\Upsilon_{f,2}(q_f, \dot{q}_f, q_{f,m}, \dot{q}_{f,m}) \in \mathbb{R}^p$ represent model uncertainty vectors of the $f$th follower. We assume that the motion of leaders is independent of that of followers, the followers 1 to $N$ have at least one neighbor, and the leaders $N+1$ to $M$ have no neighbors. Notice that all FJ followers (1) can have heterogeneous dynamics.

For the $N+M$ agents, the adjacent matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{(N+M)\times(N+M)}$ related with the digraph $\mathcal{G}$ is defined as $a_{ij} > 0$ if $(j,i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise where $i = 1, \ldots, N+M$ and $j = 1, \ldots, N+M$. Self-edges are not allowed, i.e., $a_{ii} = 0$. Notice that $a_{ij} = 0, i = N+1, \ldots, N+M, j = 1, \ldots, N+M$ since the leaders have no neighbors. The (nonsymmetric) Laplacian matrix $\mathcal{L}$ is represented by $\mathcal{L} = \mathcal{D} - \mathcal{A} \in \mathbb{R}^{(N+M)\times(N+M)}$ where $\mathcal{D} = \mathrm{diag}[d_1, \ldots, d_{N+M}]$; $d_i = \sum_{j=1, j\neq i}^{N+M} a_{ij}$ is the diagonal element of the degree matrix $\mathcal{D}$. To describe the communication among the followers and the communication between the followers and the leaders separately, the Laplacian matrix $\mathcal{L}$ can be rewritten as

$$\mathcal{L} = \begin{bmatrix} \overline{\mathcal{L}}_1 & \overline{\mathcal{L}}_2 \\ 0_{M\times N} & 0_{M\times M} \end{bmatrix} \tag{2}$$

where $\overline{\mathcal{L}}_1 \in \mathbb{R}^{N\times N}$ is the matrix related to the communication among the $N$ followers and $\overline{\mathcal{L}}_2 \in \mathbb{R}^{N\times M}$ denotes the matrix related to the communication from the $M$ leaders to the $N$ followers.

**Definition 1.** Boyd and Vandenberghe, 2004. The set $\mathcal{X} \subseteq \mathbb{R}^n$ is said to be convex if for any $x_1, x_2 \in \mathcal{X}$ and any $\theta \in [0,1]$, the point $\theta x_1 + (1-\theta)x_2$ is in $\mathcal{X}$. The convex hull $\mathrm{Co}(X)$ for a set of points $X = \{x_1, \ldots, x_n\}$ in $\mathcal{X}$ is the minimal convex set containing all points in $X$ and is defined as $\mathrm{Co}(X) = \{\sum_{i=1}^n \theta_i x_i | x_i \in X, \theta_i > 0, \sum_{i=1}^n \theta_i = 1\}$.

*The objective of this paper* is to design distributed adaptive containment control laws $u_f$ for the FJ followers (1) with unmatched uncertainties so that under the directed communication graph, the follower outputs $y_f(t)$ converge to the convex hull spanned by the dynamic leaders $r_l(t), l = N+1, \ldots, N+M$, i.e., $\inf_{h(t)\in R(t)} \|y_f(t) - h(t)\| < \epsilon$ where $f = 1, \ldots, N, R(t) = \mathrm{Co}\{r_{N+1}, \ldots, r_{N+M}\}$, and $\epsilon$ is a positive constant which can be made sufficiently small, while all signals in the total closed-loop systems are bounded.

We use the following properties and assumptions throughout this paper.

**Property 1.** Lewis, Abdallah, and Dawson, 1993. *The inertia matrix $M_f(q_f)$ is a symmetric and uniformly bounded positive definite matrix.*

**Property 2.** Lewis et al., 1993. *The Coriolis-centripetal matrix $C_f(q_f, \dot{q}_f)$ can be defined such that the matrix $\dot{M}_f(q_f) - 2C_f(q_f, \dot{q}_f)$ is the skew-symmetric matrix.*

**Assumption 1.** The functions $\Upsilon_{f,1}(q_f, \dot{q}_f, q_{f,m})$ and $\Upsilon_{f,2}(q_f, \dot{q}_f, q_{f,m}, \dot{q}_{f,m})$ representing the unstructured model uncertainties are bounded and *unknown* on the digraph $\mathcal{G}$ where $f = 1, \ldots, N$.

**Assumption 2.** The multiple dynamic leaders $r_l(t) \in \mathbb{R}^p$, $l = N+1, \ldots, M$ are bounded and available for only the $f$th followers satisfying $l \in \mathcal{N}_f(t), f = 1, \ldots, N$. In addition, their first derivatives $\dot{r}_l(t) \in \mathbb{R}^p$ are bounded as $\|\dot{r}_l(t)\| \leqslant \bar{r}_{l,0}$ where $\bar{r}_{l,0}$ are *unknown* constants.