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Bootstrap control charts in monitoring value at risk in insurance

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ABSTRACT

A risk measure is a mapping from the random variables representing the risks to a number. It is estimated using historical data and utilized in making decisions such as allocating capital to each business line or deposit insurance pricing. Once a risk measure is obtained, an efficient monitoring system is required to quickly detect any drifts in the risk measure. This paper investigates the problem of detecting a shift in value at risk as the most widely used risk measure in insurance companies. The probabilistic C control chart and the parametric bootstrap method are employed to establish a risk monitoring scheme in insurance companies. Since the number of claims in a period is a random variable, the proposed method is a variable sample size scheme. Monte Carlo simulations for Weibull, Burr XII, Birnbaum–Saunders and Pareto distributions are carried out to investigate the behavior and performance of the proposed scheme. In addition, a real example from an insurance company is presented to demonstrate the applicability of the proposed method.

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1. Introduction

A risk measure attempts to assign a numerical value to a random functional loss. Risk measurement is used as an input in many decisions such as the amount of holding capital for an insurance company or prices of different types of insurance services. There are several risk measures in the literature, however, the most widely used risk measure in insurance is value at risk (VaR). Wang et al. (2005) present the applications of VaR in insurance companies and discuss on optimal insurance contracts. VaR_p (also denoted by ξ_p) is the pth quantile of the loss distribution, or it can be defined as the size of loss for which there is a small probability $1-p$ of exceeding. For risk managers it is very important to detect (quickly) in which areas of their insurance business there is a "deviation" from what could be considered as a "normal" activity. If VaR is at 99%, then 1 out of 100 losses is expected to be larger than VaR. However, in day-to-day operations, we may observe less or more than 1 loss out of 100 losses, then it is vital to know whether the observed deviation is a random error or it is due to an assignable cause which necessitates a change in decisions. The control chart is a useful aid, frequently used in the process control, to discriminate the effects of assignable causes versus the effects of chance causes. In insurance companies, a control chart can be used to alarm when too many and so the risk manager should do something to control the claims.

In mathematical form $\mathrm{VaR}_p(X)$ is:

$$\mathrm{VaR}_p(X) \equiv \inf\{x|Pr(X > x) \leqslant 1-p\} \quad (1)$$

where p is called confidence level and often is selected 0.95 or 0.99 in practice. X denotes the random variable that refers to the loss size. We will drop sub-index p reference to X for an easier notation. Although VaR has been criticized for not being a coherent risk measure as introduced by Artzner et al. (1999) since it is not sub-additive in general but it is classified into natural risk statistics introduced by Heyde et al. (2007), it is known as a robust risk measure with respect to modeling assumptions. For a justification of the concept and a comprehensive study of VaR we refer to Jia and Dyer (1996) and Krause (2003). Ma and Wong (2010) establish some behavior foundations for various types of VaR models and discuss several alternative risk measures for investors. It is well known that loss data usually have right-skewed distributions (Bolancé et al., 2003, 2008; Buch-Larsen et al., 2005; Lane, 2000). Fig. 1 shows a histogram of third party claims severity data borrowed from an insurance company which exhibits a typical right-skewed behavior with many small claims and only a few large claims. There are two approaches to estimate the distribution of loss data. In the first approach, the data are fitted to a distribution with high flexibility in shape such as Weibull, Gamma, Pareto or Burr XII distributions; this approach is called the parametric approach. Another approach is called the non-parametric approach which uses empirical distribution or kernel function to estimate the probability density function

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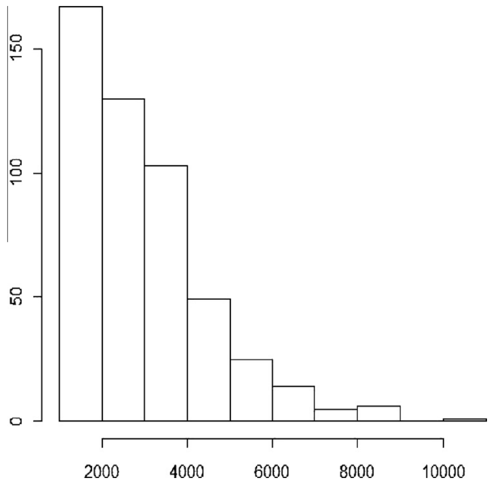

Fig. 1. Histogram of the third party claims (n = 500).

of data. The parametric approach in loss distribution analysis is widely used in practice.

After estimating VaR, it is important to monitor and detect any shifts in the risk.[2] In statistical quality control terminology, estimating VaR using historical data can be interpreted as phase I of statistical process control whereas we estimate the parameters of population distribution and establish the control limits. Mihailescu (2004) applies an Exponentially Weighted Moving Average (EWMA) control chart in monitoring VaR when the loss random variable follows a normal distribution. However, normality assumption is not reliable for the loss function in the most of the cases (Bolancé et al., 2003). Detecting shifts enables insurance companies to revisit their policies and decisions to prevent from enormous losses. In this article we design bootstrap control charts with variable sample size to monitor VaR in insurance companies. Moreover, we employ a probabilistic C control chart in monitoring the number of claims in a specific horizon of time. Hence, In phase I we establish two control charts: (1) a probabilistic C chart for monitoring the number of claims (C) in each period and (2) a control chart to monitor VaR based on claim values (X) in each period. To construct a control chart for a parameter, the distribution of the parameter estimator is required otherwise the bootstrapping should be used. In monitoring VaR, since the distribution of the VaR estimator is unknown we implement bootstrap control charts. Moreover, since the sample size is the number of claims in each period, which is a random variable, the designed monitoring scheme is a variable sample size bootstrap scheme.

The remainder of this paper is organized as follows. Section 2 gives an overview on the methods to estimate VaR when data are fitted to one of the following distributions: Weibull, Burr XII, Birnbaum–Saunders or Pareto distributions. Section 3 describes bootstrap methods to establish control limits for quantiles. The monitoring scheme for the number of claims and the severity of claims is explained in Section 4. The performance evaluation using extensive simulation studies considering in-control average run length (ARL) and out-of-control ARL is discussed in Section 5. A real example from an insurance company is illustrated in Section 6. Finally, Section 7 offers conclusions and final remarks.

[2] Risk monitoring and customer auditing is essential to the guarantee solvency in the insurance industry (see recent contributions by Guelman, 2012; Kaishev et al., 2013; Koyuncugil and Ozgulbas, 2012; Shin et al., 2012; Thuring et al., 2012).

2. Estimating value at risk

The estimation of VaR is equal to estimating a quantile of a population, and the estimation is subject to errors. Stephens (1983) shows the nonparametric estimation of VaR is biased. For the parametric approach, Kupiec (1995) in a simulation study using return distributions that are normal or Student-t, indicates that the estimation of VaR is subject to both high variation and bias. In order to take the bias into consideration, there are some bias corrected methods. For instance Efron (1982) estimates bias, b, by $\hat{b} = \Phi^{-1}(\hat{\xi}_{p,boot})/\hat{\sigma}_{\xi_p,boot}$ where $\hat{\xi}_{p,boot}$ and $\hat{\sigma}_{\xi_p,boot}$ are bootstrap estimates of pth quantile and its standard error respectively. The standard error of VaR_p estimator typically increases when p increases and the behavior of a quantile estimate is critically altered by the tail of the loss distribution. Despite of the large standard error of VaR, it is known as a robust risk measure. Kendall and Stuart (1972) derive formula for the asymptotic variance of quantile estimator: for ξ_p (pth quantile) of X with density $f_X(x)$, this variance is:

$$\sigma^2_{\xi_p,n} = n^{-1}p(1-p)f_X(\xi_p)^{-2} + O(n^{-3/2}) \quad (2)$$

where n is the sample size. There are two principle approaches in estimating VaR from the historical data. The first approach is fitting a parametric distribution to the data, in this approach a distribution that takes a variety of shapes such as Weibull or Burr XII distribution is fitted to the data and the goodness-of-fit test is employed to check the model fit. Then VaR as a quantile of the distribution is obtained from the best fitted distribution. For an instance of this approach we refer to Sun and Hong (2010), who apply importance sampling in the parametric method to estimate VaR. The second approach is using the non-parametric density estimation techniques that fits a non-parametric distribution based on the empirical distribution or the kernel function to the historical data; one can then estimate VaR from non-parametric density function. Chang et al. (2003) and Jeong and Kang (2009) use the non-parametric method to estimate VaR. Zmeskal (2005) proposes a method to estimate VaR in fuzzy environments. If the empirical distribution function is used in a non-parametric approach the VaR_p is obtained by one of the following formulas:

$$\hat{\xi}_p^{p,method1} = X_{(\lfloor (n-1)p \rfloor + 1)} \quad (3)$$

$$\hat{\xi}_{p,method2} = X_{(\lfloor (n+1)p \rfloor)} \quad (4)$$

$$\hat{\xi}_{p,method3} = X_{(\lfloor (n-1)p \rfloor + 1)} + ((n-1)p - \lfloor (n-1)p \rfloor)(X_{(\lfloor (n-1)p \rfloor + 2)} - X_{(\lfloor (n-1)p \rfloor + 1)}) \quad (5)$$

$$\hat{\xi}_{p,method4} = X_{(\lfloor (n+1)p \rfloor)} + ((n+1)p - \lfloor (n+1)p \rfloor)(X_{(\lfloor (n+1)p \rfloor + 1)} - X_{(\lfloor (n+1)p \rfloor)}) \quad (6)$$

where $\lfloor . \rfloor$ is the floor function and $X_{(i)}$ is the ith order statistic of X in the sample. Another method to estimate pth quantile is the jackknife method. The delete-one jackknife estimate of ξ_p is obtained by:

$$\hat{\xi}_{p,jackknife} = (\lfloor (n-1)p \rfloor + 1)n^{-1}X_{(\lfloor (n-1)p \rfloor + 2)} + (1 - (\lfloor (n-1)p \rfloor + 1)n^{-1})X_{(\lfloor (n-1)p \rfloor + 1)} \quad (7)$$

and the jackknife estimate of the variance of $\hat{\xi}_{p,jackknife}$ is given by (see Martin, 1990; Miller, 1974):

$$\hat{\sigma}_{\xi_{p,jackknife}} = (n-1)(n - \lfloor (n-1)p \rfloor + 1)(\lfloor (n-1)p \rfloor + 1)n^{-2} (X_{(\lfloor (n-1)p \rfloor + 2)} - X_{(\lfloor (n-1)p \rfloor + 1)})^2 \quad (8)$$

In the appendix we explore the biases of the above estimations. Martin (1990) shows that the variance of the jackknife estimator of sample quartile is inconsistent. Shao (1987) obtains a consistent

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