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Identification of continuous-time Hammerstein systems by simultaneous perturbation stochastic approximation

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ABSTRACT

This paper proposes an identification method for Hammerstein systems using simultaneous perturbation stochastic approximation (SPSA). Here, the structure of nonlinear subsystem is assumed to be unknown, while the structure of linear subsystem, such as the system order, is assumed to be available. The main advantage of the SPSA-based method is that it can be applied to identification of Hammerstein systems with less restrictive assumptions. In order to clarify this point, piecewise affine functions with a large number of parameters are adopted to approximate the unknown nonlinear subsystems. Furthermore, the linear subsystems are supposed to be described in continuous-time. Though this class of systems closely reflects the actual systems, there are few methods to identify such models. Hence, the SPSA-based method is utilized to identify the parameters in both linear and nonlinear subsystems simultaneously. The effectiveness of the proposed method is evaluated through several numerical examples. The results demonstrate that the proposed algorithm is useful to obtain accurate models, even for high-dimensional parameter identification.

1. Introduction

The modelling of real-world plants and processes, which are nonlinear in nature, remains a challenging problem. Both an expert and intelligent systems are therefore required to model accurately such plants and processes. One way to cope with this difficulty is to introduce identification of block oriented models. These models include a Hammerstein model (a static nonlinear subsystem followed by a linear subsystem), a Wiener model (a linear subsystem followed by a static nonlinear subsystem), or a Hammerstein–Wiener model (a linear subsystem sandwiched by two static nonlinear subsystems or vice-versa). In particular, an intelligent system, such as a system with a neural feed-forward controller, is modeled as a Hammerstein model. These models have been adopted by many researchers partly because they closely reflect actual nonlinear systems with relatively simple structures. As a result, these models have been successfully used to describe many practical plants, such as fuel cells (Li, Zhu, Cao, Sui, & Hu, 2008), valve actuators (Wang & Zhang, 2014), wind turbines (van der Veen, van Wingerdeen, & Verhaegen, 2013), spark ignition engine torques (Togun, Baysec, & Kara, 2012), and stirred tank reactors (Shi, Xu, & Dai, 2011).

Among various types of nonlinear system models, the Hammerstein model is quite popular. In fact, the identification of Hammerstein systems has been widely reported in the literature (Bai, 2004; Bai & Li, 2004, Ding & Chen, 2005; Ding, Liu, & Liu, 2011; Ding, Shi, & Chen, 2006; 2007b; Greblicki, 2000; Hasiewicz & Mzyk, 2004; Liu & Bai, 2007; Pawlak, 1991; Zhao, 2006). Over the past two decades, various methods for identification of Hammerstein systems have been studied extensively. These can be roughly classified into several categories, such as the iterative method (Liu & Bai, 2007; Narendra & Gallman, 1966; Rangan, Wolodkin, & Polla, 1995; Stoica, 1981; Voros, 1997), the over-parameterization method (Chang & Luus, 1971; Ding, Chen, & Iwai, 2007a; Hsia, 1976), the blind approach (Bai & Fu, 2002), the subspace method (Verhaegen & Westwick, 1996), the least squares method (Ding & Chen, 2005; Goethals, Pelckmans, Suykens, & Moor, 2005), the parametric instrumental variables method (Laurain, Gilson, & Garnier, 2009; Stoica & Soderstrom, 1981), the stochastic method (Bilings & Fakhouri, 1978; Greblicki, 1996; Pawlak, 1991) and the non-parametric identification method (Bai, 2003; Greblicki & Pawlak, 1987; Krzyak, 1993, 1996).

Recently, a decomposition-based Newton iterative identification approach for a Hammerstein nonlinear FIR system with ARMA noise was presented by Ding, Deng, and Liu (2014). Here, it was claimed that a fast convergence rates with more accurate parameter estimation can be achieved by using the Newton iterative method. In

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Wang, Ding, and Ximei (2015), a hierarchical least squares method has been proposed for the identification of Hammerstein state space model. A similar approach is applied to Hammerstein nonlinear controlled autoregressive models (Chen & Ding, 2015). Both results show that the hierarchical identification principle may improve the computational efficiency by decomposing one nonlinear system into several subsystems with smaller dimensions and fewer variables. In Ma and Liu (2015), a nonlinear recursive instrumental variables (RIV) identification method for Hammerstein ARMAX system is adopted. The effectiveness of the RIV method is shown in terms of identification accuracy and convergence speed, especially under colored noise. Meanwhile, a blind approach with new over-sampling strategy was adopted in Yu, Zhang, and Xie (2014) to produce a consistent parameter estimation in the presence of noise.

In spite of such abundant literature, several restrictions are inevitable in their studies from the theoretical view-point.

(i) Most of the identification methods are restricted to the models in discrete-time, while it is natural to express a real system in a continuous-time domain.
(ii) Many approaches assume that the static nonlinear system is given by a linear combination of several basis functions.
(iii) In the over-parameterization identification method, the identification model contains the products of nonlinear and linear parameters, causing redundant parameter identification and a large computational load.

Though it looks that that we can handle general class of nonlinear subsystems by adopting so many basis functions (such as higher order polynomials and piecewise affine functions), this is not tractable in reality in the existing identification frame work.

On the other hand, there are different types of identification methods which utilize evolutional computation, such as the cuckoo search algorithm (Gotmare, Patidar, & George, 2015), the stochastic gradient (Chen & Wang, 2015; Mao & Ding, 2015), and the PSO (particle swarm optimization) (Jingzhuo et al., 2014; Ko, 2011; Nanda, Panda, & Majhi, 2010; Wang, Ren, Liu, & Han, 2014). These methods are quite flexible in nature, and do not suffer from (i) and (iii) mentioned above. In particular, the PSO is known to be effective in various systems control supplication (Maruta, Kim, Song, & Sugie, 2013; Maruta, Kim, & Sugie, 2009). However, they have a serious draw back.

(iv) In swarm based optimization (including PSO), the computation times per iteration are proportional to the number of swarms. As a result, these methods require heavy computation time in the identification process, especially for static nonlinear systems with a large number of basis functions.

Hence, it is not tractable to handle static nonlinear subsystems consisting of large number of basis. Consequently, these evolutional computation based methods cannot avoid (ii) as well.

To the best of the authors' knowledge, the simultaneous perturbation stochastic approximation (SPSA) method (Spall, 1992) could be only candidate to provide us with a promising tool for such system identification problems. This is because the SPSA method is well known to be effective for a variety of optimization problems, even for high-dimensional parameter tuning (Ahmad, Azuma, & Sugie, 2014b). In comparison to the existing results, the SPSA method does not suffer from the afore-mentioned theoretical restrictions (i)–(iv). It may be expected to identify both linear and nonlinear subsystems simultaneously, even for large number of basis functions with less computational load. Meanwhile, one major drawback of SPSA may be to guarantee the local convergence only from the theoretical points of view.

Based on the above observations, this paper thus presents an identification method of Hammerstein systems in continuous-time using simultaneous perturbation stochastic approximation. We assume that the structure (i.e., the system order) of the linear subsystem is known in advance, while the structure of the nonlinear subsystem remains unknown. Here, a piecewise affine function is then used to approximate the unknown nonlinear function in the Hammerstein model. Next, based on the input and output data, the SPSA-based method is used to identify the coefficients of the linear time-invariant (LTI) model and the piecewise affine function simultaneously. In order to clarify the benefit of the SPSA-based method, a large number of parameters in the piecewise affine function are considered here. So far, there have been few papers discussing the identification of such Hammerstein models. Therefore, it is worth evaluating the effectiveness of the SPSA method.

The remainder of this paper is organized as follows. Section 2 formulates the identification problem for Hammerstein models. In Section 3, the identification method using simultaneous perturbation stochastic approximation-based algorithm is presented. The nonlinear function identification based on piecewise affine function is also described in the same section. The method is validated through several numerical examples in Section 4. Finally, some concluding remarks are given in Section 5. This paper is based on our preliminary version (Ahmad, Azuma, & Sugie, 2014a), published in a conference proceedings, and contains the full explanations and experiments omitted there.

Notation: The symbols $\mathbb{R}$ and $\mathbb{R}_+$ represent the set of real numbers and the set of positive real numbers, respectively. For the vector $\boldsymbol{\theta}$, we use $\|\boldsymbol{\theta}\|_2$ to express the standard Euclidean norm. For $\delta \in \mathbb{R}_+$, $\mathrm{sat}_\delta : \mathbb{R}^n \to \mathbb{R}^n$ denotes the saturation function whose ith element given as follows:

$$\text{The } i\text{-th element of } \mathrm{sat}_\delta(\boldsymbol{x}) = \begin{cases} \delta & \text{if } \delta < x_i, \\ x_i & \text{if } -\delta \le x_i \le \delta, \\ -\delta & \text{if } x_i < -\delta \end{cases}$$

where $x_i \in \mathbb{R}$ is the ith element of $\boldsymbol{x} \in \mathbb{R}^n$.

2. Problem formulation

Consider the continuous-time single-input-single-output (SISO) Hammerstein model in Fig. 1, composed of a nonlinear function f and a linear dynamical system G described by the differential operator p $(:= \frac{d}{dt})$:

$$G(p) = \frac{B(p)}{A(p)} = \frac{b_m p^m + b_{m-1} p^{m-1} + \cdots + b_0}{p^n + a_{n-1} p^{n-1} + \cdots + a_0}. \tag{1}$$

Here, $u(t)$ is the input, $\bar{u}(t)$ is the unmeasurable output of the nonlinear function, namely $\bar{u}(t) = f(u(t))$, $\tilde{y}(t)$ is the measurement of $y(t)$ but is corrupted by the noise $v(t)$. The input–output relationship is described as follows:

$$\tilde{y}(t) = G(p) f(u(t)) + v(t). \tag{2}$$

In this paper, we address an identification problem of the Hammerstein model. Here, we assume that:

- m and n are known.
- $a_i (i = 0, 1, \ldots, n-1)$ and $b_i (i = 0, 1, \ldots, m)$ are positive real numbers.
- The function f is unknown, but $f(u(t))$ is a one-to-one map to $u(t)$. Moreover, $f(0) = 0$.

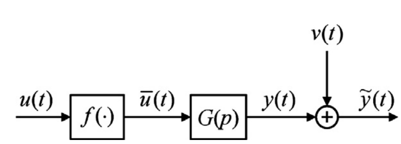

Fig. 1. The continuous-time SISO Hammerstein model.