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A cooperative coevolutionary algorithm for the Multi-Depot Vehicle Routing Problem

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ABSTRACT

The Multi-Depot Vehicle Routing Problem (MDVRP) is an important variant of the classical Vehicle Routing Problem (VRP), where the customers can be served from a number of depots. This paper introduces a cooperative coevolutionary algorithm to minimize the total route cost of the MDVRP. Coevolutionary algorithms are inspired by the simultaneous evolution process involving two or more species. In this approach, the problem is decomposed into smaller subproblems and individuals from different populations are combined to create a complete solution to the original problem. This paper presents a problem decomposition approach for the MDVRP in which each subproblem becomes a single depot VRP and evolves independently in its domain space. Customers are distributed among the depots based on their distance from the depots and their distance from their closest neighbor. A population is associated with each depot where the individuals represent partial solutions to the problem, that is, sets of routes over customers assigned to the corresponding depot. The fitness of a partial solution depends on its ability to cooperate with partial solutions from other populations to form a complete solution to the MDVRP. As the problem is decomposed and each part evolves separately, this approach is strongly suitable to parallel environments. Therefore, a parallel evolution strategy environment with a variable length genotype coupled with local search operators is proposed. A large number of experiments have been conducted to assess the performance of this approach. The results suggest that the proposed coevolutionary algorithm in a parallel environment is able to produce high-quality solutions to the MDVRP in low computational time.

1. Introduction

A Vehicle Routing Problem (VRP) is a generic name for a large class of combinatorial optimization problems (Doerner & Schmid, 2010; Montoya-Torres, Franco, Isaza, Jiménez, & Herazo-Padilla, 2015). The goal is to find a set of routes for serving customers with a certain number of vehicles in a given environment. In the classical VRP, a problem instance is specified by a set of customers to be served with their corresponding locations and demands and other primary information such as distance between two costumers, distance between a customer and the depot, number of vehicles and vehicle capacity (Baldacci & Mingozzi, 2009). In a solution, each vehicle leaves the depot and executes a route over a certain number of customers before returning to the depot, while insuring that the total demand on the route does not exceed vehicle capacity. In some cases, a maximum route duration (or distance) constraint is enforced. The Multi-Depot Vehicle Routing Problem (MDVRP) is a variant of the classical VRP in which more than one depot is considered (Cordeau & Maischberger, 2012; Escobar, Linfati, Toth, & Baldoquin, 2014; Subramanian, Uchoa, & Ochi, 2013; Vidal, Crainic, Gendreau, Lahrichi, & Rei, 2012).

The number of studies on the MDVRP is rather limited when compared to the classical VRP. A survey of these studies, based on either exact methods or heuristics, can be found in Montoya-Torres et al. (2015). In recent years, evolutionary-based metaheuristics proved to be a popular approach to address this problem, as described in Section 3. But, in spite of this popularity, no coevolutionary algorithm has yet been proposed in the literature for the MDVRP. As the problem can be easily decomposed into a number of single-depot VRPs, with a

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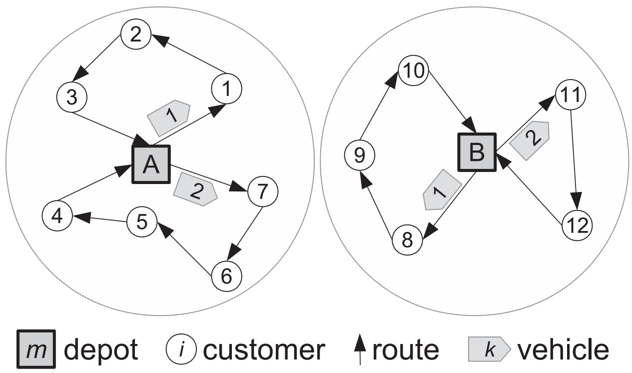

Fig. 1. MDVRP solution.

population of partial solutions associated with each depot, a coevolutionary approach looks relevant. Each partial solution or individual in a population corresponds to vehicle routes defined over the subset of customers assigned to the corresponding depot. Although each population can evolve separately, this evolution is guided by the ability of each individual to form good complete solutions with individuals from the other populations. This is the problem-solving approach proposed in this work.

The remainder of this paper is organized as follows. First, some preliminaries about the MDVRP and coevolution are found in Section 2. Section 3 presents a literature review. Sections 4 and 5 describe the proposed methodology while Section 6 reports computational results. Future avenues for research are proposed in the conclusion in Section 7.

2. Preliminaries

In this section, some preliminary information about the mathematical formulation of the MDVRP and cooperative coevolutionary algorithms are presented.

2.1. Multi-Depot Vehicle Routing Problem formulation

As mentioned earlier, the MDVRP is a variant of the classical VRP where more than one depot is considered (Montoya-Torres et al., 2015). Fig. 1 shows a typical solution of this problem with two depots and two vehicle routes associated with each depot. Typically, the fleet of vehicles is limited and homogeneous (Cordeau & Maischberger, 2012; Escobar et al., 2014; Montoya-Torres et al., 2015; Subramanian et al., 2013; Vidal et al., 2012).

Basically, a solution to this problem is a set of vehicle routes such that: (i) each vehicle route starts and ends at the same depot, (ii) each customer is served exactly once by one vehicle, (iii) the total demand on each route does not exceed vehicle capacity (iv) the maximum route time is satisfied and (v) the total cost is minimized (Montoya-Torres et al., 2015).

The MDVRP can be formalized as follows. Let $G = (V, A)$ be a complete graph, where V is the set of nodes and A is the set of arcs. The nodes are partitioned into two subsets: the customers to be served, $V_C = \{1, \ldots, N\}$, and the multiple depots $V_D = \{N+1, \ldots, N+M\}$, with $V_C \cup V_D = V$ and $V_C \cap V_D = \emptyset$. There is a non-negative cost c_{ij} associated with each arc $(i, j) \in A$. The demand of each customer is d_i (there is no demand at the depot nodes). There is also a fleet of K identical vehicles, each with capacity Q. The service time at each customer i is t_i while the maximum route duration time is set to T. A conversion factor w_{ij} might be needed to transform the cost c_{ij} into time units. In this work, however, the cost is the same as the time and distance units, so $w_{ij} = 1$.

In the mathematical formulation that follows, binary variables x_{ijk} are equal to 1 when vehicle k visits node j immediately after node i. Auxiliary variables y_i are also used in the subtour elimination constraints.

$$\text{Minimize} \sum_{i=1}^{N+M} \sum_{j=1}^{N+M} \sum_{k=1}^{K} c_{ij} x_{ijk}, \tag{1}$$

subject to:

$$\sum_{i=1}^{N+M} \sum_{k=1}^{K} x_{ijk} = 1 \ (j = 1, \ldots, N); \tag{2}$$

$$\sum_{j=1}^{N+M} \sum_{k=1}^{K} x_{ijk} = 1 \ (i = 1, \ldots, N); \tag{3}$$

$$\sum_{i=1}^{N+M} x_{ihk} - \sum_{j=1}^{N+M} x_{hjk} = 0 \ (k = 1, \ldots, K; \ h = 1, \ldots, N+M); \tag{4}$$

$$\sum_{i=1}^{N+M} \sum_{j=1}^{N+M} d_i x_{ijk} \le Q \ (k = 1, \ldots, K); \tag{5}$$

$$\sum_{i=1}^{N+M} \sum_{j=1}^{N+M} \left(c_{ij} w_{ij} + t_i\right) x_{ijk} \le T \ (k = 1, \ldots, K); \tag{6}$$

$$\sum_{i=N+1}^{N+M} \sum_{j=1}^{N} x_{ijk} \le 1 \ (k = 1, \ldots, K); \tag{7}$$

$$\sum_{j=N+1}^{N+M} \sum_{i=1}^{N} x_{ijk} \le 1 \ (k = 1, \ldots, K); \tag{8}$$

$$y_i - y_j + (M+N) x_{ijk} \le N + M - 1; \quad \text{for } 1 \le i \ne j \le N \text{ and } 1 \le k \le K; \tag{9}$$

$$x_{ijk} \in \{0, 1\} \ \forall \ i, j, k; \tag{10}$$

$$y_i \in \{0, 1\} \ \forall \ i; \tag{11}$$

The objective (1) minimizes the total cost. Constraints (2) and (3) guarantee that each customer is served by exactly one vehicle. Flow conservation is guaranteed through constraint (4). Vehicle capacity and route duration constraints are found in (5) and (6), respectively. Constraints (7) and (8) check vehicle availability. Subtour elimination constraints are in (9). Finally, (10) and (11) define x and y as binary variables.

In the original formulation of the MDVRP, a fixed number of vehicles is allocated to each depot. In our work, though, the search is allowed to consider a larger number of vehicles (at a penalty cost in the objective). This is discussed in Section 5.

2.2. Coevolutionary algorithms

Coevolutionary algorithms are a class of evolutionary algorithms inspired by the simultaneous evolution process involving two or more species. Recently, various engineering problems have been solved with this approach (Blecic, Cecchini, & Trunfio, 2014; Chen, Mori, & Matsuba, 2014; Ladjici & Boudour, 2011; Ladjici, Tiguercha, & Boudour, 2014; Wang & Chen, 2013a; Wang, Cheng, & Huang, 2014). Coevolutionary algorithms are categorized into two groups depending on the type of interaction among the species, which can be either competitive or cooperative. *Competitive coevolution* can be viewed as an *arms race*, that is, individuals in the populations compete among themselves. One group attempts to take advantage over another, which responds with an adaptive strategy to recover the advantage (Katada & Handa, 2010). A biological example is the *predator–prey* competitive coevolution, in which the evolution of one population