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Solving the winner determination problem via a weighted maximum clique heuristic

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ABSTRACT

Combinatorial auctions (CAs) where bidders can bid on combinations of items is an important model in many application areas. CAs attract more and more attention in recent years due to its relevance to fast growing electronic business applications. In this paper, we study the winner determination problem (WDP) in CAs which is known to be NP-hard and thus computationally difficult in the general case. We develop a solution approach for the WDP by recasting the WDP into the maximum weight clique problem (MWCP) and solving the transformed problem with a recent heuristic dedicated to the MWCP. The computational experiments on a large range of 530 benchmark instances show that the clique-based approach for the WDP not only outperforms the current best performing WDP heuristics in the literature both in terms of solution quality and computation efficiency, but also competes very favorably with the powerful CPLEX solver.

1. Introduction

Combinatorial auctions (CAs) is a type of auctions where bidders are allowed to buy entire bundles of goods (or items) in a single transaction (Cramton, Shoham, & Steinberg, 2006). One key issue in CAs is the winner determination problem (WDP) (Lehmann, Rudolf, & Sandholm, 2006). Given a set of combinatorial bids, each bid being defined by a subset of items with a price, two bids are conflicting if they share at least one item. The WDP is to determine a conflict-free allocation of items to bidders (the auctioneer can keep some of the items) that maximizes the auctioneer's revenue. It is known that the WDP is equivalent to the maximum weight set packing problem (de Vries & Vohra, 2003), and can be reduced to the maximum weight clique problem (MWCP). From the computational complexity point of view, the WDP belongs to the class of NP-complete problems (Rothkopf, Pekeč, & Harstad, 1998). From the practical point of view, the WDP finds many applications in, for instance, production management (Ray, Jenamani, & Mohapatra, 2011), intelligent transportation systems (Satunin & Babkin, 2014; de Vries & Vohra, 2003), electronic commerce (de Vries & Vohra, 2003), game theory (Fontanini & Ferreira, 2014), knowledge management (Wu, 2001), logistics services (Ignatius, Lai, Motlagh, Sepehri, & Mustafa, 2011; Pla, López, Murillo, & Maudet, 2014; de Vries & Vohra, 2003).

The WDP can be reduced to the maximum vertex weight clique problem (MWCP) (Ausiello, D'Atri, & Protasi, 1980). As a consequence, any solution method designed for the MWCP can be applied to solve the WDP via its clique formulation. This solution approach is extremely appealing since (1) we can solve the WDP without developing dedicated WDP algorithms, and (2) we can take full advantage of new algorithmic developments on the MWCP to better solve the WDP. Moreover, one can even apply different clique methods to enlarge the classes of the WDP instances that can be solved. As far as we know, this clique based approach for the WDP was not explored in the published literature.

The first objective of the paper is thus to investigate the strong connection between the WDP and the MWCP by carrying out an in-depth experimental assessment about the performance of this clique-based approach for the WDP. For this purpose, we adopt the recent multi-neighborhood tabu search heuristic (MN/TS) for the MWCP (Wu, Hao, & Glover, 2012) and present extensive evaluations of this approach for the WDP both in terms of solution quality and computing efficiency. In particular, we provide computational results on three sets of well-known WDP test suites (for a total of 500 + 20 + 10 = 530 problem instances) which are commonly used in the literature. We show that this clique-based approach is clearly superior to the current best performing heuristics in the

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literature which are specially designed for the WDP. Moreover, it dominates the powerful CPLEX 12.4 solver on the realistic test suite and shows a competitive performance on the other two test suites.

The rest of the paper is organized as follows. In Section 2, we provide a review on existing algorithms for the WDP and summarize the main contributions of this work. In Section 3, we present the formal definition of the WDP and the transformation of the WDP to the maximum weight clique problem. In Section 4, we briefly recall the multi-neighborhood tabu search heuristic MN/TS for the MWCP. In Section 5, we provide computational results and comparisons on a wide range of benchmark instances from the literature. In Section 6, we offer some insights on the behavior of the clique approach. The last section is dedicated to conclusions and perspectives for future research.

2. Literature review and main contributions

The computational challenge of the WDP and its wide practical applications have motivated a variety of solution approaches, including both exact and heuristic methods.

Exact methods have the theoretical advantage of guaranteeing the optimality of the solution found, but they need a computing time which grows exponentially with the problem size in the general case. Still, highly effective exact algorithms are available in the literature for solving the WDP. Attempts to exactly solve the WDP (under the name of set packing) can be found as early as in the beginning of 1970s (Padberg, 1973). Many studies have appeared ever since. Most exact algorithms are based on the general branch-and-bound (B&B) framework. Representative examples include the combinatorial auction structural search (CASS) (Fujishima, Leyton-Brown, & Shoham, 1999), the Combinatorial Auction Multi-Unit Search (CAMUS) (Leyton-Brown, Shoham, & Tennenholz, 2000; Leyton-Brown, 2003), the BOB algorithm (Sandholm & Suri, 2003), the CABOB algorithm (Sandholm, Suri, Gilpin, & Levine, 2005), the linear programming based B&B algorithm (Nisan, 2000), and the clique-based B&B algorithm using graph coloring for bounding (Wu & Hao, 2014). Other interesting exact methods for the WDP are a branch-and-price algorithm based on a set packing formulation (Günlük, Lászlo, & de Vries, 2005), a branch-and-cut algorithm (Escudero, Landete, & Marín, 2009), and a dynamic programming algorithm (Rothkopf et al., 1998). Finally, the general integer programming approach based on CPLEX was intensively studied in (Andersson, Tenhunen, & Ygge, 2000; Guo, Lim, Rodrigues, & Zhu, 2006; Sandholm et al., 2005), showing an excellent performance in many cases.

On the other hand, given the intrinsic intractability of the WDP, various heuristic algorithms have been devised to handle problems whose optimal solutions cannot be reached by exact approaches. For instance, Casanova (Hoos & Boutilier, 2000) is a well-known stochastic local search algorithm which explores the space of feasible allocations (non-overlapping subsets of bids) by adding at each step an unallocated bid and removing from the allocation the bids which are conflicting with the added bid. The selection rule employed by Casanova takes into consideration of both the quality and history information of the bid. Casanova is shown to be able to find high quality solutions much faster than the CASS algorithm (Fujishima et al., 1999). The WDP is also modeled as a set packing problem and solved by a simulated annealing algorithm (SAGII) with three different local move operators: an embedded branch-and-bound move, greedy local search move and exchange move (Guo et al., 2006). SAGII outperforms dramatically Casanova and the CPLEX 8.0 solver for realistic test instances. A memetic algorithm is proposed by Boughaci, Benhamou, and Drias (2009), which combines a local search component with a specific crossover operator. The local search component adds at each iteration either a random bid with a probability p or a best bid with the largest profit with probability $1-p$, and then removes the conflicting bids from the allocation. This hybrid algorithm reaches excellent results on the tested realistic instances. Other interesting heuristics include greedy algorithms (Lau & Goh, 2002; Mito & Fujita, 2004), a tabu search algorithm (Boughaci, Benhamou, & Drias, 2010), an equilibrium-based local search method (Tsung, Ho, & Lee, 2011) and a recombination-based tabu search algorithm (Sghir, Hao, Ben Jaafar, & Ghédira, 2014).

From the above review, we observe that the existing (exact and heuristic) methods follow two solution strategies. The first one is to consider directly the WDP and design dedicated algorithms. This is the case for most of the reviewed methods. The second one is to recast the WDP as another related problem P and then solved with a solution method designed for P. Examples are shown in Guo et al. (2006) and Padberg (1973) where the WDP is modeled as the set packing problem and in Andersson et al. (2000), Guo et al. (2006) and Sandholm et al. (2005) where the WDP is reformulated as an integer programming problem and solved by the general CPLEX solver.

Compared with the existing studies on the WDP, this work has the following main contributions:

First, we handle the WDP by recasting it as a weighted maximum clique problem and applying an effective clique heuristic to solve the problem. To our knowledge, this is the first study formally investigating the strong connection between the WDP and the MWCP and presenting extensive computational assessments of the clique based approach to the WDP.

Second, this study discloses that the clique based approach is well suited for the WDP, and is able to delivery very competitive and even better results than the current best performing WDP heuristics which are specially designed for the problem. This is particularly true for the cases where each bid contains a relatively long list of item.

Third, from a more technical perspective, it is well known that move operators play a key role to the performance of a local search algorithm. Most of the currently best WDP heuristics rely only on a basic 'add-and-repair' operator which first adds an unallocated bid to the current allocation and then removes the conflicting bids from the allocation. From the clique point of view, this basic move operator is quite limited and effective clique algorithms employ more complicated operators like add, swap and drop (Pullan, 2006; Pullan & Hoos, 2006; Wu & Hao, 2014). This study indirectly demonstrates the usefulness of these combined move operators for the WDP problem, promoting the idea that to design effective WDP heuristics, it would be relevant to integrate similar combined operators.

3. Winner determination problem (WDP)

The optimal winner determination problem in CAs can be defined as follows. Let $M = \{1, 2, \ldots, m\}$ be the set of m items to be sold by the auctioneer, and let $B = \{B_1, B_2, \ldots, B_n\}$ be the set of bids submitted by the buyers. Each bid can be denoted by a couple (S_i, P_i), where $S_i \subseteq M$ is a set of items and P_i is the global price of the items in S_i. Let B be a $m \times n$ binary matrix such that $B_{ij} = 1$ if object $j \in S_i, B_{ij} = 0$ otherwise. Furthermore, define $x_i = 1$ if the bid B_i is accepted (a winning bid), and zero otherwise (a losing bid). Then the winner determination problem (WDP) is to label the bids as winning or losing so as to maximize the auctioneer's revenue under the constraint that each item can be allocated to at most one bidder. More formally, the WDP problem can be modeled as the following integer programming formulation.