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Memetic search for the quadratic assignment problem

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ABSTRACT

The quadratic assignment problem (QAP) is one of the most studied NP-hard problems with various practical applications. In this work, we propose a powerful population-based memetic algorithm (called BMA) for QAP. BMA integrates an effective local optimization algorithm called Breakout Local Search (BLS) within the evolutionary computing framework which itself is based on a uniform crossover, a fitness-based pool updating strategy and an adaptive mutation procedure. Extensive computational studies on the set of 135 well-known benchmark instances from the QAPLIB revealed that the proposed algorithm is able to attain the best-known results for 133 instances and thus competes very favorably with the current most effective QAP approaches. A study of the search landscape and crossover operators is also proposed to shed light on the behavior of the algorithm.

1. Introduction

The quadratic assignment problem (QAP) is a classic NP-hard combinatorial optimization problem with a number of applications (Cheng et al., 2012; Garey and Johnson, 1979; Li et al., 2012; Miao et al., 2014; Nikolić and Teodorović, 2014; Pardalos et al., 1994). QAP is to determine a minimal cost assignment of n facilities to n locations, given a flow a_{ij} from facility i to facility j for all $i, j \in \{1, \ldots, n\}$ and a distance b_{qp} between locations q and p for all $q, p \in \{1, \ldots, n\}$. Let Π denote the set of the permutation functions $\pi : \{1, \ldots, n\} \to \{1, \ldots, n\}$, then QAP can mathematically be formulated as follows:

$$\min_{\pi\in\Pi} f(\pi) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} b_{\pi_i \pi_j} \tag{1}$$

where a and b are the flow and distance matrices respectively, and $\pi \in \Pi$ is a solution where π_i represents the location chosen for facility i. The problem objective is then to find a permutation π^* in Π that minimizes the sum of the products of the flow and distance matrices, i.e., $f(\pi^*) \leqslant f(\pi), \ \forall \pi \in \Pi$.

Besides the facility location problem, QAP is notable for its ability to formulate a number of other practical problems such as backboard wiring in electronics, design of typewriter keyboards, campus planning, analysis of chemical reactions for organic compounds, balancing turbine runners, and many others. QAP can equally formulate some classic combinatorial optimization problems such as the traveling salesman, maximum clique and graph partitioning problems. Reviews on some significant applications of QAP can be found in Burkard (1991), Duman and Or (2007) and Pardalos et al. (1994), while many solution methods are reviewed in Anstreicher (2003).

QAP is among the most studied and the hardest combinatorial optimization problems. In fact, from a theoretical point of view, QAP is NP-hard (Garey and Johnson, 1979). Consequently, no exact algorithm is expected to solve the problem in a polynomial time and even small instances may require considerable computation time. This hardness is confirmed in practice since the existing exact algorithms can solve to optimality only small instances from the QAP benchmark library with up to 36 locations. Even approximation of the problem with a guaranteed performance is known to be very hard (Hassin et al., 2009). For these reasons, heuristic and metaheuristic methods constitute a natural and useful approach for tackling this problem (Blum et al., 2011). Such algorithms aim to provide satisfactory sub-optimal solutions in acceptable computing time, but with no theoretically provable guarantee that the attained solutions are the optimal ones. Performance of these heuristic algorithms is typically assessed using a set of benchmark instances. Among the numerous heuristic algorithms reported for QAP in the literature, local search methods are very popular approaches, including simulated annealing (Wilhelm and Ward, 1987), tabu search (Battiti and Tecchiolli, 1994, 2009a, 2009b, 2006, 1990, 1991), and iterated local search (Benlic and Hao, 2013c; Stützle, 2006). Population-based approaches

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constitute another class of popular tools for finding high quality near-optimal solutions for QAP (Ahuja et al., 2000; Drezner, 2003; Drezner, 2008; Fleurent and Ferland, 1993; Merz and Freisleben, 2000; Misevicius, 2004; Stützle, 2006).

In this work, we are interested in solving QAP with heuristic algorithms. We introduce a powerful memetic algorithm (BMA) for QAP which combines an effective local search algorithm (BLS – Breakout Local Search), a crossover operator, a pool updating strategy, and an adaptive mutation mechanism. BLS is a general local search method which has shown very good results for several NP-hard problems including maximum clique (Benlic and Hao, 2013a), maximum cut (Benlic and Hao, 2013b), QAP (Benlic and Hao, 2013c) and vertex separator (Benlic and Hao, 2013d). Its basic idea is to use a descent procedure to discover local optima and employ dedicated perturbations to continually move from one attractor to another in the search space. In this paper, we integrate BLS into the memetic framework, thus extending our work of Benlic and Hao (2013c). As we show in this paper, the proposed memetic algorithm BMA exhibits an excellent performance on the whole set of 135 well-known QAP benchmark instances. Indeed, BMA attains the best-known solution for all the instances except for only two cases. Furthermore, it outperforms its local search procedure BLS which confirms the usefulness of the memetic framework. In order to gain some insight into the functioning of the proposed memetic algorithm, we perform a landscape analysis and justify the choice for the used crossover operator (see Section 5).

The paper is organized as follows. In the next section, we briefly review the most effective QAP approaches and highlight the contributions of this work. In Section 3, we present our proposed memetic algorithm (BMA) and detail its main components. Moreover, we highlight the differences and similarities between BMA and the reviewed state-of-art approaches. Computational results and comparisons with the top-performing QAP algorithms are presented in Section 4. Section 5 provides a landscape study that we use to justify the choices for the crossover operator of our memetic algorithm, and additionally shows a comparison of several crossover operators. Conclusions are provided in the last section.

2. State-of-art approaches for QAP and main contributions

In this section, we provide a literature review of the most popular heuristic approaches for QAP, including four population-based algorithms (Drezner, 2008; Merz and Freisleben, 2000; Misevicius, 2004; Stützle, 2006) and two local search algorithms (James et al., 2009a; Misevicius and Kilda, 2006), followed by a summary of the main contributions of this study. In Section 3.5, we discuss in more detail the relationships between these approaches and the proposed BMA. Among the reviewed approaches, four algorithms from Drezner (2008), James et al. (2009a), Misevicius (2004) and Misevicius and Kilda (2006) report the best results on some particular class of QAP benchmark instances. For this reason, these four algorithms will be used as reference algorithms for our comparative study. Nevertheless, it is important to mention that none of the existing QAP approaches can be considered as the most effective for all the different types of QAP instances, due to significant differences in structure of these instance (see Section 5).

A popular memetic approach for QAP (MA-QAP) is introduced in Merz and Freisleben (2000) which incorporates the 2-opt local search procedure and an adaptation of the standard uniform crossover UX that does not perform any implicit mutation. The selection for reproduction is performed on a purely random basis, while the selection for survival is achieved by choosing the best individuals from the pool of parents and children. To overcome premature convergence, the restart technique proposed by Eshelman (1991) is employed. During the run, it is checked whether the average distance of the population has dropped below a certain threshold or whether the average fitness of the population did not change after a certain number of consecutive generations. If one of these conditions holds, the whole population is mutated except the best individual, and each mutated individual is improved by the 2-opt local search to obtain a local optimum. Afterwards, the algorithm proceeds with the usual recombination process. We mention this work since it is one of the first memetic algorithms applied to QAP and achieved remarkable results at the time it was published.

The Improved Hybrid Genetic Algorithm (IHGA) (Misevicius, 2004) incorporates a robust local improvement procedure as well as an effective restart mechanism based on shift mutations. The author slightly improved the classic scheme of a uniform like crossover (ULX) to get a new optimized crossover (OX). The optimized crossover is a crossover that (a) is ULX and (b) produces a child that has the best fitness value among the children created by M runs of ULX. The offspring is then improved with a local search mechanism, based on the swap neighborhood, which contains a tabu search procedure and a solution reconstruction procedure. The reconstruction is achieved by performing a number μ of random swaps, where μ is varied according to the instance size. Once convergence of the algorithm is observed, all the individuals but the best undergo the shift mutation (SM), which simply consists in shifting all the items in a wrap-around fashion by a predefined number of positions. IHGA is one of the best-performing algorithms for the unstructured instances and real-life like instances and is used as one of the references in our comparative study.

Drezner (2008) shows extensive computational experiments on QAP using various variants of a hybrid genetic algorithm. The author compared the modified robust tabu search (MRT) and the simple tabu search as local optimization algorithms combined with a crossover operator. Moreover, different parent selection (distance modification, gender modification) and pool updating strategies were tested. The best version of the memetic algorithm is MRT60 which integrates the modified robust tabu search MRT for offspring improvement. MRT is identical to the robust tabu search (RTS) (Taillard, 1991) except that the tabu tenure is generated in $[0.2\,n, 1.8\,n]$ rather than in $[0.9\,n, 1.1\,n]$. MRT60 is the best-performing memetic algorithm for the grid-based instances and is used as another reference algorithm in our comparative study.

Population-based Iterated Local Search (PILS) (Stützle, 2006) is another highly effective algorithm. The underlying iterated local search (ILS) algorithm starts from a random assignment, and applies a first-improvement local search procedure based on the 2-opt neighborhood. To speed up the search process, the algorithm uses the *do not look bit* strategy, previously proposed to accelerate local search algorithms for TSP. Once a local optimum is reached, ILS applies a perturbation that consists of exchanging k randomly chosen items, where k is varied between k_{min} and k_{max}. This ILS algorithm is extended to a population-based algorithm where no interaction between solutions takes place, and each single solution is improved by ILS. In the proposed PILS, the population consists of μ solutions and in each iteration λ new solutions are generated. A selection strategy, based both on quality and distance between solutions, is then employed to form a new population of μ solutions from the set of $\mu + \lambda$ solutions.

Cooperative parallel tabu search algorithm (CPTS) (James et al., 2009a) executes in parallel several tabu search (TS) operators on multiple processors. The TS operator is a modified version of Taillard's RTS (Taillard, 1991) obtained by changing the stopping criterion and the tabu tenure parameters for each processor participating in the algorithm. In order to accomplish cooperation between TS processes, CPTS maintains a global reference set which uses information exchange to promote both intensification and diversification in a parallel environment. CPTS globally obtains excellent results on the whole set of QAP instances and is used as another reference algorithm in our comparative study.