Expert Systems with Applications 41 (2014) 2409–2414

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

New capability indices for measuring the performance of a multidimensional machining process

Jeh-Nan Pan [a,*], Chung-I Li [b]

[a] Department of Statistics, National Cheng Kung University, Tainan 70101, Taiwan, ROC
[b] Department of Applied Mathematics, National Chiayi University, Chiayi 60004, Taiwan, ROC

ARTICLE INFO

Keywords:
Process capability indices
Engineering tolerance
Positional tolerance
Geometric dimensioning and tolerancing
Multidimensional machining process

ABSTRACT

Engineering tolerance plays an important role in the process capability analysis for determining whether a manufacturing process is capable of making good quality products. In contrast with the engineering tolerance region in a multivariate manufacturing process, the multidimensional machining process or the nano-cutting process has a special engineering tolerance called the positional tolerance. Positional tolerance is a special type of geometric dimensioning and tolerancing which describes the tolerance region between the actual location of machining results and the target location. In the past few years, several capability indices have been developed for measuring the performance of a multidimensional machining process under the assumption that the variances of machining results on different directions are equal. However, this assumption may not be true in most practical situations. In this paper, we propose three novel capability indices for measuring the performance of a multidimensional machining process under the assumption that the variances of machining results on different directions may not be equal. The statistical properties of the point estimators and their confidence intervals for the new capability indices are derived. Both the simulation results and numerical examples show that the new capability indices outperform the predecessors.

1. Introduction

Process capability analysis is a very important SPC tool for monitoring and evaluating process performance. In the past two decades, many researchers and practitioners have devoted their efforts to the task of developing suitable capability indices for various manufacturing processes. Engineering tolerance plays an important role in the process capability analysis for determining whether a manufacturing process is capable of making good quality products. Geometric dimensioning and tolerancing (GD&T), on the other hand, is an engineering standard (ANSI Y14.5M-1994) providing a unified terminology and methodology for describing both the geometry of product features and their associated tolerances. Following these principles, the GD&T tolerance zone for the location of a hole is a circle circumscribing the square tolerance zone (i.e. positional tolerances). Positional tolerance is a special type of engineering tolerance which describes the tolerance region between the actual location of machining results and the target location. In contrast with the engineering tolerance region in a multivariate manufacturing process, the multidimensional machining process or the nano-cutting process has a specific specification called the positional tolerance.

To measure positional performance for a multidimensional machining process, Krishnamoorthi (1990) proposed PC_p and PC_{pk} indices that are extensions of the C_p and C_{pk} indices. Assuming the process mean is equal to the target, Davis, Kaminsky, and Saboo (1992) proposed an index $R = U/\sigma$, where U is the radius of specification region and σ is the standard deviation of quality characteristic. Karl, Morisette, and Taam (1994) extended the concept of the multivariate process capability proposed by Taam, Subbaiah, and Liddy (1993). Moreover, Bothe (2006) considered the radial distance between the target and the actual hole location as a quality characteristic to assess the capability of a process by locating the hole centers within a circular tolerance zone. Note that the above process capability indices for measuring the performance of a two or three dimensional machining process are developed under the assumption that the variances of machining results on different directions are equal. However, this assumption may not be true in most practical cases. For example, Jackson (2006) gave a practical example of a two dimensional machining process, in which the variances of machining results on different directions (i.e. X or Y axis) are unequal. In addition, most modern nano-cutting processes as shown in the second numerical example, their variances of machining results on different directions may not be equal either. If a multidimensional machining process is mistreated as a

* Corresponding author. Tel.: +886 6 2757575; fax: +886 6 2342469.
E-mail address: jnpan@mail.ncku.edu.tw (J.-N. Pan).

0957-4174/$ - see front matter
http://dx.doi.org/10.1016/j.eswa.2013.09.039

multivariate manufacturing process, it will result in an improper decision and thereby lead to an unnecessary quality loss. Thus, it is necessary to develop new capability indices for measuring the performance of a multidimensional machining process under the assumption of unequal variances.

2. Literature review

2.1. Process capability index

Process capability indices have been widely used in industry to provide quantitative measures of process performance that lead to quality improvement. The most commonly used process capability indices are:

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

$$C_{pk} = \min\left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right) \tag{2}$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{E[(X-T)^2]}} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} \tag{3}$$

$$C_a = 1 - \frac{|\mu - (USL + LSL)/2|}{(USL - LSL)/2} = 1 - \frac{|\mu - m|}{d} \tag{4}$$

$$C_{pmk} = \min\left(\frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right) \tag{5}$$

where μ is the process average, σ is the process standard deviation, USL is upper specification limit, LSL is lower specification limit $m = (USL + LSL)/2$ and T is the target value. Juran (1974) proposed the C_p index. It considers the ratio of the engineering tolerance to the natural tolerance, thus it reflects only the process precision. The C_{pk} index proposed by Kane (1986) considers both the process precision and the process accuracy. Considering the loss function approach, the C_{pm} index proposed by Chan, Cheng, and Spiring (1988) adds an additional penalty for process shift, that is, as the mean drifts away from the target. Pearn, Kotz, and Johnson (1992) proposed C_{pmk} index which is more sensitive to the actual performance of the population than C_p, C_{pk}, or C_{pm} as the process mean deviates from the target. To measure the degree of process centering, Pearn, Lin, and Chen (1998) proposed C_a as the process accuracy index. Lin and Pearn (2003) further pointed out that the mathematical relationship can be established among the indices in Eqs. (1), (2), and (4) as

$$C_{pk} = C_p(1 - C_a) \tag{6}$$

2.2. Process capability index for the positional tolerance

In the past two decades, the topic of developing process capability index for the positional tolerance has been extensively discussed in many multidimensional machining problems. Krishnamoorthi (1990) pointed out that the process capability indices mentioned in Section 2.1 becomes inadequate for measuring performance of a manufacturing process when positional tolerances are specified. In order to measure the performance of a manufacturing process with positional tolerance, Krishnamoorthi (1990) proposed PC_p and PC_{pk} indices that are extensions of the C_p and C_{pk} indices. Assuming the process mean is equal to the target, Davis et al. (1992) showed that the non conforming rates for two-dimensional and three-dimensional cases are

$$\exp\left(-\frac{(U/\sigma)^2}{2}\right) \tag{7}$$

and

$$1 - \left\{\Phi\left(\frac{U/\sigma}{\sqrt{2}}\right) - \frac{U}{\sigma}\sqrt{\frac{2}{\pi}}\exp\left(-\frac{(U/\sigma)^2}{2}\right)\right\}. \tag{8}$$

They further proposed an index $R = U/\sigma$, where U is the radius of specification region and σ is the standard deviation of quality characteristic. To measure the performance of a manufacturing process with positional tolerance, Karl et al. (1994) extended the concept of the multivariate process capability proposed by Taam et al. (1993). Moreover, Bothe (2006) considered the radial distance between the target and the actual hole location as a quality characteristic to assess the capability of a process by locating the hole centers with in a circular tolerance zone. Tahan and Cauvier (2012) developed an explicit mathematical model to identify the distribution functions (PDF and CDF) of defects on the location and diameter. They used those distributions and the Hasofer–Lind index to propose a new process capability index.

3. Development of process capability index for positional tolerance

Assuming a multidimensional machining process follows a multivariate normal distribution, the tolerance region for a manufacturing process with the spherical tolerance can be written as:

$$(X_1 - t_1)^2 + \cdots + (X_p - t_p)^2 \le U^2, \tag{9}$$

where $(X_1, X_2, \ldots, X_p)$ is the actual location of machining results, U is the radius of specification and $(t_1, t_2, \ldots, t_p)$ is the target location. Suppose that $(X_1, X_2, \ldots, X_p)$ are independent, then the average squared distance between the actual location and the target location is given by

$$E((X_1 - t_1)^2 + \cdots + (X_p - t_p)^2) = \sum_{i=1}^{p}(\mu_i - t_i)^2 + \sum_{i=1}^{p}\sigma_i^2, \tag{10}$$

where μ_i and σ_i are the process mean and standard deviation of the ith quality characteristic, respectively. The average squared distance, on the left hand side of Eq. (10), can be treated as a measure of performance for a manufacturing process with positional tolerance. As one can see in Eq. (10), average squared distance can be divided into two parts; the first term on the right hand side of Eq. (10) is a measure of the process accuracy and the second term on the right hand side of Eq. (10) is a measure of the process precision. In order to properly evaluate the performance of process accuracy, we propose the following process capability index:

$$NPC_a = \frac{\sum_{i=1}^{p}(\mu_i - t_i)^2}{U^2} \tag{11}$$

Since the value of the NPC_a index is large (small) as the distance between the process mean and the target location is large (small), the index NPC_a can provide the information concerning process accuracy. To measure the performance of process precision, we utilize the concept proposed by Pan and Lee (2010) and propose the following index:

$$NPC_p = \frac{U^2}{c_p \sum_{i=1}^{p}\sigma_i^2}, \tag{12}$$

where $c_p = (\chi^2_{p,0.9973})^{\frac{p}{2}} \Big/ p$ and $\chi^2_{p,0.9973}$ is the α percentile of chi-square distribution with p degrees of freedom. Note that $c_1 = 2.9997$, $c_2 = 5.9145$, and $c_3 = 17.7542$.