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A linear ordering on the class of Trapezoidal intuitionistic fuzzy numbers

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ABSTRACT

Fuzzy numbers and intuitionistic fuzzy numbers are introduced in the literature to model problems involving incomplete and imprecise information in expert and intelligent systems. Ranking of TrIFNs plays an important role in an information system (Decision Making) with imprecise and inadequate information and the complete ranking on the class of trapezoidal intuitionistic fuzzy number is an open problem worldwide. Researchers from all over the world have been working in ranking of intuitionistic fuzzy numbers since 1985, but till date there is no common methodology that ranks any two arbitrary intuitionistic fuzzy numbers due to the partial ordering of TraIFNs. Different algorithms are available in the literature for solving intuitionistic fuzzy decision (or information system) problem, but each and every algorithm failed to give better result in some places due to the ranking procedure of TrIFNs. Intuitionistic fuzzy decision algorithm works better when it have a complete ranking procedure that ranks arbitrary intuitionistic fuzzy numbers. In this paper a linear (total) ordering on the class of trapezoidal intuitionistic fuzzy numbers using axiomatic set of eight different scores is introduced. The main idea of this paper is to classify and study the properties of eight different sub classes of the set of TrIFNs. Further new total order relations are defined on each of the subclasses of TrIFNs and they are extended to a complete ranking procedure on the set of TrIFNs. Finally the significance of the proposed method over existing methods is studied by illustrative examples.

1. Introduction

Any expert and intelligent information system involve information collected from experts. The information for the most part, may contain the mix of subjective, quantitative and deficient data procured from the specialists. Despite the fact that the specialists express their assessment with the best of their insight, they might likewise have absence of information about requirements, in such cases the data got from specialists about an item may be inadequate. These kind of issues can be demonstrated more precisely using intuitionistic fuzzy sets (Atanassov, 1986) than fuzzy sets. Intuitionistic fuzzy sets are utilized to model such incomplete data by the membership, non membership and hesitancy degree. So reality estimation of fragmented data is given by (μ, ν, π) with $\mu + \nu + \pi = 1$ which implies that data is valid with grade μ, not valid with grade ν and inadequate with grade π.

Diverse ranking methodologies available in the literature for fuzzy and intuitionistic fuzzy information systems are based on dominance classes and the entire dominance degree, which have been studied in Du, Hu, and Zhao (2011), Geetha, Lakshmana Gomathi Nayagam, and Ponalagusamy (2012), Song, Liang, and Qian (2012), Xu (2007), Xu and Chen (2007), Xu and Yager (2006), Zhang and Mi (2004).

Ranking of IVIFN assumes a vital part in problems with inadequate information since the membership, non-membership and hesitance degree are better to represent in intervals.

Geetha et al. (2012), portrayed intuitionistic fuzzy interval information system utilizing dominance degree as a part of perspective of the ranking of intuitionistic fuzzy interval numbers as described in Lakshmana Gomathi Nayagam, Muralikrishnan, and Geetha (2011). When compared with interval valued intuitionistic fuzzy number (IVIFN), trapezoidal intuitionistic fuzzy number (TrIFN) is considered to be an important class of data in decision making analysis. Unclear and uncertain information can be dealt with ideal using TrIFN over IVIFN. By stretching out intuitionistic fuzzy interval numbers to TrIFNs we are given the capacity to handle unclear and uncertain information successfully, because the imprecise, membership and non-membership degrees are only

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needed to be expressed as linguistic terms rather than ranges of values. So the study on the incomplete trapezoidal information system is inexorable for tackling issues, in actuality, applications.

Mitchell (2004), has introduced a new ranking method for fuzzy number by adopting statistical viewpoint and interpreting each intuitionistic fuzzy number (IFN) as an aggregation of ordinary fuzzy number. Lakshmana Gomathi Nayagam, Venkateshwari, and Geetha (2008), defined a new ranking method for triangular intuitionistic fuzzy number (TIFN) which generalizes the scoring method introduced by Chen and Hwang.

Li (2010), presented the thought of value and ambiguity of a triangular intuitionistic fuzzy number and presented a ranking method utilizing the idea of the proportion of the value index to the ambiguity index. Ye (2011), introduced another new ranking method using expected value of a trapezoidal intuitionistic fuzzy number and tackled the decision making problem using weighted expected value of TrIFN. Dubey and Mehara (2011), extended the idea of value and ambiguity to the marginally adjusted TIFN and proposed a new approach to solve intuitionistic fuzzy linear programing problem. Nehi (2010), presented the idea of characteristic values of membership and non-membership elements of TrIFN and proposed another ranking method for trapezoidal intuitionistic fuzzy numbers by utilizing it. Zhang and Nan (2013), built up a compromise ratio ranking method for fuzzy multi attribute decision making (MADM) problem with an idea that bigger TIFN among different TIFNs will be closer to the greatest value index and it will be far from the base ambiguity index all the while. Kumar and Kaur (2013), proposed a ranking method for TrIFNs by altering nehi's method (2010). Zeng, Li, and Yu (2014), presented a new ranking method for TrIFNs by broadening the idea value and ambiguity of TIFN characterized in Li (2010). Wana and Dong (2015), introduced the thought of lower and upper weighted possibility mean and possibility mean for a trapezoidal intuitionistic fuzzy numbers and proposed a new ranking method by use of it. Ranking of TrIFNs using countable number of parameters was studied in Lakshmana Gomathi Nayagam, Jeevaraj, and Geetha (2016).

The main idea of this paper is to classify and study the properties of eight different sub-classes of the set of TrIFNs. In this paper, new total order relations on each of the subclasses of TrIFNs are defined and a linear ordering on the class of trapezoidal intuitionistic fuzzy numbers using axiomatic set of total order relations defined on the sub-classes is introduced. Further the limits and illogicalities of all the aforementioned methods are examined and the effectiveness of our proposed technique is demonstrated by looking at all current methods using numerical examples.

1.1. Preliminaries

Some essential definitions are given in this section.

Definition 1.1.1. (Burillo, Bustince, Mohedano, & Lakov, 1994)

An intuitionistic fuzzy number $A = (\mu_A, \nu_A)$ in the set of real numbers $\Re$, is defined as

$$\mu_A(x) = \begin{cases} f_A(x) & \text{if } a \le x \le b_1 \\ 1 & \text{if } b_1 \le x \le b_2 \\ g_A(x) & \text{if } b_2 \le x \le c \\ 0 & \text{Otherwise} \end{cases}$$

and $\nu_A(x) = \begin{cases} h_A(x) & \text{if } e \le x \le f_1 \\ 0 & \text{if } f_1 \le x \le f_2 \\ k_A(x) & \text{if } f_2 \le x \le g \\ 1 & \text{Otherwise} \end{cases}$ where $0 \le \mu_A(x) + \nu_A(x) \le 1$

and $a, b_1, b_2, c, e, f_1, f_2, g \in \Re$ such that $e \le a$, $f_1 \le b_1 \le b_2 \le f_2$, $c \le g$, and four functions $f_A, g_A, h_A, k_A: \Re \to [0, 1]$ are the legs of membership function μ_A and nonmembership function ν_A. with the functions f_A and k_A are nondecreasing continuous functions and the functions h_A and g_A are non-increasing continuous functions and is denoted by $\{(a, b_1, b_2, c), (e, f_1, f_2, g)\}$.

An intuitionistic fuzzy number $\{(a, b_1, b_2, c), (e, f_1, f_2, g)\}$ is shown in Fig (1).

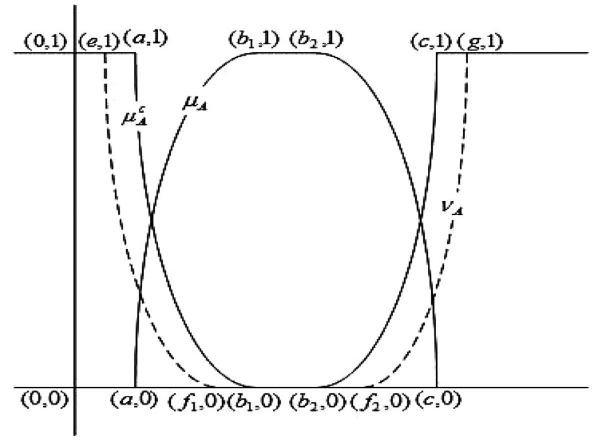

Fig. 1. Intuitionistic fuzzy number.

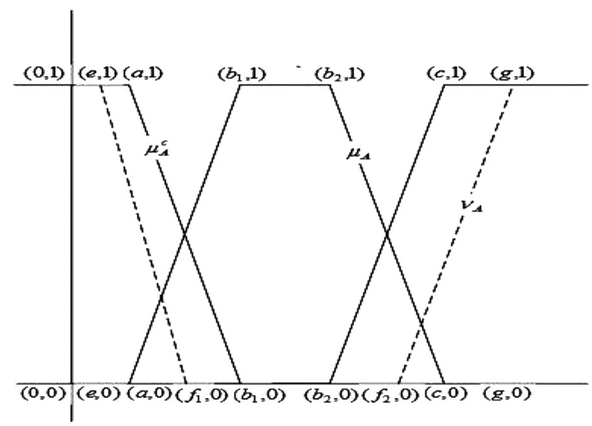

Fig. 2. Trapezoidal intuitionistic fuzzy number.

Definition 1.1.2. A trapezoidal intuitionistic fuzzy number A with parameters $e \le a$, $f_1 \le b_1 \le b_2 \le f_2$, $c \le g$, is denoted as $A = \{(a, b_1, b_2, c), (e, f_1, f_2, g)\}$ in the set of real numbers $\Re$ is an intuitionistic fuzzy number whose membership function and non-membership function are given as

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{if } a_1 \le x \le a_2 \\ 1 & \text{if } a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3} & \text{if } a_3 \le x \le a_4 \\ 0 & \text{Otherwise} \end{cases}$$

and $\nu_A(x) = \begin{cases} \frac{x-c_2}{c_1-c_2} & \text{if } c_1 \le x \le c_2 \\ 0 & \text{if } c_2 \le x \le c_3 \\ \frac{x-c_3}{c_4-c_3} & \text{if } c_3 \le x \le c_4 \\ 1 & \text{Otherwise} \end{cases}$ If $a_2 = a_3 (\text{and} c_2 = c_3)$ in a trapezoidal intuitionistic fuzzy number A, we have the triangular intuitionistic fuzzy numbers as special case of the trapezoidal intuitionistic fuzzy numbers .

A trapezoidal intuitionistic fuzzy number $A = \{(a, b_1, b_2, c), (e, f_1, f_2, g)\}$ with $f_1 \le b_1$, $f_2 \ge b_2$, $e \le a$, and $g \ge c$ is shown in fig (2).

Definition 1.1.3. (Atanassov and Gorgov, 1989). Let $D[0, 1]$ be the set of all closed subintervals of the interval [0, 1]. An interval valued intuitionistic fuzzy set on a set $X \neq \phi$ is an expression given