Expert Systems with Applications 40 (2013) 964–974

Contents lists available at SciVerse ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

New developments in uncertainty assessment and uncertainty management

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ARTICLE INFO

Keywords:
Uncertainty quantification
Fuzzy sets
Inference
Total uncertainty
Management

ABSTRACT

The paper presents a general method for doing predictions and test planning which can also be used as a tool for managing uncertainty. Uncertainty is generally defined as ''that which is not precisely known''. This definition permits the identification of different kinds of uncertainty arising from different sources and activities, most of which go unnoticed in analysis. In this paper, the description of uncertainty begins from a historical perspective and concludes with a new perspective based upon making inferences; fuzzy logic can be most helpful in quantifying some inferences. Our assessment of uncertainty begins the identification of the various forms of uncertainty (ambiguity, fuzziness, randomness, non-specificity, ignorance, etc.) and concludes with models and methods for assessing the 'total uncertainty' within an application. The material contained herein is described in the context of physical science and engineering applications; however, nothing presented precludes application to other fields, e.g. economics, social sciences, medicine and business. Uncertainty assessment involves how to identify, classify, characterize, quantify, and combine uncertainties within an application, with the expressed goal of understanding how to manage uncertainties. Uncertainty management presumes that we have a process to quantify uncertainties and to be able to aggregate them in such a way that they can be compared in terms of their individual contributions to the 'total uncertainty'. Managing uncertainties is important, because uncertainties directly affect decision and policy making. An example, using a concept called Quantification of Margins and Uncertainty (QMU), is provided to illustrate our ideas.

1. Introduction

We have spent many decades trying to understand and quantify uncertainty. The more uncertainty in a problem, the less precise or correct we can be in our understanding of that problem. Most science and engineering endeavors do not address the uncertainty in the information, models, and solutions that are conveyed within the problem. We propose that the characterization and quantification of uncertainty within such problems should be done commensurate with what is known or can be determined in the physical world and with an appropriate level of expressed precision. One reason for engaging in such a pursuit is obvious: achieving high levels of precision costs significantly in time, money or both. The more complex a problem or system is, the more imprecise or inexact is the information that we have to characterize that system and hence the greater the uncertainty about it. Thus, uncertainty is related to precision, information, and complexity, making its assessment paramount in the problems we pose for eventual solution. Lotfi Zadeh has a famous quote about the importance of balancing the precision we seek with the uncertainty that exists (Zadeh, 1973), ''*we must exploit our tolerance for imprecision*''.

This paper suggests that uncertainty of various forms permeates all scientific endeavors, and it exists as an integral feature of all abstractions, models, theory, and solutions. It is our intent to summarize methods to handle some of these forms of uncertainty in our technical problems. Since much of what we have explored in the past 45 years is new and not part of a canonical jargon, we begin first with the definition of some terms and some critical thinking about uncertainty.

2. Some thoughts and definitions

The discussion of the assessment and management of uncertainty begins with some definitions and concepts:

Certainty can operationally be defined as ''a state such that evidence to the contrary is below a threshold of disputation.'' For example, if we say that initial conditions in an experiment are known with certainty, we are understood as saying that initial conditions are indisputably fixed. This definition of certainty coincides with *determinism* or a deterministic solution.

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http://dx.doi.org/10.1016/j.eswa.2012.05.054

Precision refers to abilities in making good predictions, being exact, being correct, maintaining control, operating within specifications, and representing the physical world. These achievements usually coincide with a high degree certainty.

Uncertainty is broadly defined as "what is not known precisely," and manifests itself in numerous ways, most of which are undetected or considered too difficult to assess. A partial list of theories used to assess uncertainty, developed over the past 45 years (beyond probability theory which is over 400 years old!), is provided below in Section 4.

Quantification (as in *uncertainty quantification* or *UQ*) refers to an analysis and assessment process or evaluation based upon models, data, expertise, etc. Quantification does not necessarily require a numerical statement or conversion to a number. Some might distinguish linguistic statements describing uncertainty from quantification by identifying the former as an uncertainty assessment. We understand that numbers, like linguistics, have an interpretation that is provided by a human, often a policy or decision-maker. Thus UQ is equally valid expressed as numbers or words. A non-numeric example of a UQ statement is: *Jet engines operate with small but stable margins.*

Confidence is a commonly used term whose definitions include words like trust, belief, reliance, and certitude; it is the state of feeling sure. "Confidence comes from repetition, from the breath of many mouths," from an anonymous quote. It is interesting to note that even the Greeks were unable to precisely (or mathematically) define what is meant by confidence. In statistics, confidence has a specific meaning in sampling and inference when referring to a *confidence interval* for an unknown parameter (e.g., the mean). The *confidence level*, $1 - \alpha$, is defined as the complement of a significance level, α, or the Type I error in statistical hypothesis testing. (Type I error is the chance, e.g., 5%, that a null hypothesis is rejected when it should not have been rejected, i.e., the null hypothesis is true. This is a chosen, and therefore, controlled error in statistical inference.) Outside of the statistical context, there is no modern day definition for the mathematical meaning or quantification of confidence. The use of the term in statistical applications notwithstanding, we propose a technical definition of confidence as having an inverse relationship to uncertainty, which is assessed and/or quantified.

Total uncertainty is the combination or aggregation of all relevant uncertainties within an application or problem. Ideally, an analyst would produce an estimate of the final or system-level overall answer to a question, y, accompanied by a total uncertainty estimate, Δy, or $y \pm \Delta y$. Constructing methods of determining total uncertainty is a relatively new area of research (see Ross, 2003).

3. Brief history of uncertainty assessment & quantification

From an historical point of view the issue of uncertainty has not always been embraced within the scientific community (Klir & Yuan, 1995). In the traditional view of science, uncertainty represents an undesirable state, a state that must be avoided at all costs. This was the state of science until the late nineteenth century when physicists realized that Newtonian mechanics did not address problems at the molecular level. Newer methods, associated with statistical mechanics, were developed which recognized that statistical averages could replace the specific manifestations of microscopic entities, accounting for what was not precisely known (i.e., uncertainty). These statistical quantities, which summarized the activity of large numbers of microscopic entities, could then be connected in a model with appropriate macroscopic variables (Klir & Yuan, 1995). Since then, the role of Newtonian mechanics and its underlying calculus that considered no uncertainty was replaced with statistical mechanics that could be described by a probability theory—a theory which could capture a form of uncertainty arising from random processes. After the development of statistical mechanics, there has been a gradual trend in science during the past century to consider the influence of uncertainty on problems, and to do so in an attempt to make models more robust, in the sense that credible solutions are achievable and at the same time quantify the amount of uncertainty.

Of course, the leading theory in quantifying uncertainty in scientific models from the late nineteenth century until the late twentieth century had been probability theory. Probability has a long history of use, dating back to the 1500s, to the time of Cardano when gamblers recognized the rules of probability in games of chance. By the time of Newton, physicists and mathematicians were formulating different interpretations of probability consist with its axioms and operational theory. The most popular ones remaining today are the relative frequency probability and the subjectivist or personalistic probability. The latter development was based upon Rev. Thomas Bayes' (*circa 1763*) powerful theorem for conditional probabilities. Subjectivist probabilities specified that a human's *degree of belief* or *willingness to bet* was a mathematically coherent interpretation of probability within its theoretical construct.

However, the gradual evolution of the expression of uncertainty using probability theory was challenged, first in 1937 by Max Black (Black, 1937) with his studies in vagueness, then with the introduction of fuzzy sets in 1965 (Zadeh, 1965). Zadeh's work had a profound influence on the thinking about uncertainty because it challenged not only probability theory as the sole representation for uncertainty, but the very foundations upon which probability theory was based: classical binary (two-valued) logic (Klir & Yuan, 1995).

The twentieth century saw the first developments of alternatives to probability theory and to classical Aristotelian logic as paradigms to address more kinds of uncertainty than just the random kind. Jan Lukasiewicz developed a multi-valued, discrete logic (*circa* 1930). In the 1960's Arthur Dempster (Dempster, 1968) developed a *theory of evidence* which, for the first time, included an assessment of ignorance, or the absence of information. In 1965, Lotfi Zadeh introduced his seminal idea in a continuous-valued logic that he called *fuzzy set theory*. In the 1970s Glenn Shafer (Shafer, 1976) extended Dempster's work to produce a complete theory of evidence dealing with information from more than one source, and Zadeh (1973) illustrated a possibility theory resulting from special cases of fuzzy sets. Later in the 1980s other investigators showed a strong relationship between evidence theory, probability theory, and possibility theory with the use of what was called *fuzzy measures* (Klir & Wierman, 1998), and what is now being termed *monotone measures* (Ross, 2010).

4. General theories of uncertainty

Since 1965, there have been numerous developments in mathematical uncertainty theories, such as possibility theory, evidence theory, and the theory of imprecise probabilities, to name a few. These theories can be collectively referred to using the title from the Klir and Wierman (1998) book, *Generalized Information Theories* or *GITs*. A partial list of mathematical uncertainty theories includes:

Probability theory (Dempster, 1968 and Feller, 1968).
Zadeh fuzzy sets and fuzzy logic.
Possibility theory (Dubois & Prade, 1988).
Dempster–Shafer evidence theory;
Imprecise probability theory (Walley, 1991).
Random Intervals (Joslyn & Booker, 2004).