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Quantification of R-fuzzy sets

Arjab Singh Khuman[a,*], Yingjie Yang[a], Robert John[b]

[a] Centre for Computational Intelligence, De Montfort University, The Gateway, Leicester LE1 9BH, UK
[b] Automated Scheduling, Optimisation and Planning (ASAP), Nottingham University, Nottingham NG8 1BB, UK

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ABSTRACT

The main aim of this paper is to connect R-fuzzy sets and type-2 fuzzy sets, so as to provide a practical means to express complex uncertainty without the associated difficulty of a type-2 fuzzy set. The paper puts forward a significance measure, to provide a means for understanding the importance of the membership values contained within an R-fuzzy set. The pairing of an R-fuzzy set and the significance measure allows for an intermediary approach to that of a type-2 fuzzy set. By inspecting the returned significance degree of a particular membership value, one is able to ascertain its true significance in relation, relative to other encapsulated membership values. An R-fuzzy set coupled with the proposed significance measure allows for a type-2 fuzzy equivalence, an intermediary, all the while retaining the underlying sentiment of individual and general perspectives, and with the adage of a significantly reduced computational burden. Several human based perception examples are presented, wherein the significance degree is implemented, from which a higher level of detail can be garnered. The results demonstrate that the proposed research method combines the high capacity in uncertainty representation of type-2 fuzzy sets, together with the simplicity and objectiveness of type-1 fuzzy sets. This in turn provides a practical means for problem domains where a type-2 fuzzy set is preferred but difficult to construct due to the subjective type-2 fuzzy membership.

1. Introduction

The work undertaken by Yang and Hinde (2010) first proposed the notion of R-fuzzy sets, the capital 'R' distinguishes it from r-fuzzy, which was proposed by Li, Wang, and Lee (1996), yet another approach to encapsulate uncertainty. The membership value of an element of an R-fuzzy set is represented as a rough set. R-fuzzy sets are an extension of fuzzy set theory that allows for the uncertain fuzzy membership value to be encapsulated within the bounds of an upper and lower rough approximation. The lower bound contains the membership values agreed upon by all, whereas the upper bound contains membership values agreed upon by at least one. Many different variations of uncertainty exist in the real world, all of which have their own associated difficulties in exacting crisp, clear and concise information. The notion of sets in a classical sense incorporates the use of crisp boundaries, either a complete inclusion of an element or object, or complete exclusion, as was stated by Cantor (1895). A set was created with the specific role of being able to evaluate a member of either belonging, or not-belonging. As it was later understood, human nature and inferencing does not work in this way, human inferencing is not preformed in such a crisp and precise manner, everything is vague to some extent. With this realisation, the concept of a classical set did not seem a fitting synthesis for human decision making. Something else was needed, ergo, the notion of *mereology* (Lesniewski, 1929), which considered the idea of an object being partially included in a set. In mathematics, crisp understandings are needed for precise reasoning, this becomes problematic when concepts based on natural language are considered. Abstract terms with inherent vagueness and ambiguity are often used in our daily communications, therefore reasoning cannot be based solely on classical logic. This gave rise to the concept of fuzzy theory (Zadeh, 1975, 1965, 1972). Fuzzy logic adopts the mantra of mereology, whereby an element can belong to a set to some degree, inclusive of its membership function; $\mu_A(x) : \mathbb{U} \to [0, 1]$. Here the element x belongs to the set A by a returned value equal to or within the range of [0, 1].

One problem that still exists is that of deriving a crisp membership function for a standard type-1 fuzzy set, as it may involve vagueness and ambiguity, hence why there have been many extensions developed in an attempt to overcome this pitfall (Deschrijver & Kerre, 2003). Atanassov intuitionistic fuzzy sets (Atanassov, 1986), where a degree of membership and degree of non-membership are presented. Shadowed sets (Pedrycz, 1998),

* Corresponding author. Tel.: +44 116 348 6857.
E-mail addresses: Arjab.Khuman@dmu.ac.uk (A.S. Khuman), yyang@dmu.ac.uk (Y. Yang), Robert.John@nottingham.ac.uk (R. John).

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where the evaluation of a membership is scored as either (1), (0) or belonging to the shadowed region [0, 1]. Interval-valued fuzzy sets (Sambuc, 1975), where the membership of an individual element is characterised as an interval itself. Type-2 fuzzy sets (Mendel & John, 2002), where the membership function itself is a type-1 fuzzy set. These *new* approaches involve the use of intervals, multiple parameters and additional fuzzy sets to describe the uncertain membership function values of fuzzy sets. However, the shortcomings that these approaches share is that they do not recognise the difference between values within their intervals or shadow areas.

The ongoing interest in type-2 fuzzy logic as a higher order form of fuzzy logic, has received a lot of attention. The use of interval type-2 fuzzy logic and the generalised approach of type-2 fuzzy logic has garnered much interest, particularly for its ability to handle higher degrees of uncertainty. As a result, its application areas are varied, but considerable work has been undertaken in clustering, classification and pattern recognition. A thorough review of type-2 fuzzy logic applications was undertaken by Melin and Castillo (2014). The majority of the applications reviewed are based on interval-type 2 fuzzy logic, which has less associated computational overhead compared to the more computationally expensive generalised type-2 approach. As every value in the secondary grade of membership is given a membership of 1, only the foot-print-of-uncertainty is often used. It can be agreed upon that a generalised approach will indeed cater and allow for better management of handling uncertainty, compared to that of the interval type-2 approach. However, the associated complexities are often the reason that an interval type-2 approach is ultimately chosen.

As the work proposed in this paper can be seen as a bridge to cater for a generalised type-2 approach, it is noteworthy to extend a mention to some of the other current works that allow for generalised type-2 equivalence. The work proposed by Melin, Gonzalez, Castro, Mendoza, and Castillo (2014), applied the theory of alpha planes which were used to help create generalised type-2 fuzzy logic for image detection. Wagner and Hagras (2010), 2013) propose the use of z-slices as means to reduce the computational burden. Mendel, Liu, and Zhai (2009) proposed the use of alpha planes to represent type-2 fuzzy logic sets.

R-fuzzy sets tackles the problem from a different perspective, via the use of rough sets to approximate the uncertain fuzzy membership function values of a fuzzy set. By utilising the approximation that rough sets employs, R-fuzzy sets allows for the membership values of the entire populous to be included. Rough sets themselves allow for a different perspective to that of fuzzy sets with regards to uncertainty. A rough perspective is with relation to ambiguity, a lack of information, whereas a fuzzy approach is more akin to vagueness, a lack of sharp definable boundaries. As a result there have been several hybridisation between fuzzy sets and rough sets to allow for greater versatility in encapsulating uncertainty; Bodjanova (2007), Deng, Chen, Xu, and Dai (2007), Dubois and Prade (1990), Dubois (1980), Huynh and Nakamori (2005), Jensen and Shen (2008), 2009), Nanda and Majumdar (1992), Pawlak and Skowron (2007), Radzikowska and Kerre (2002), Sun, Ma, and Chen (2014), Wu, Mi, and Zhang (2003), Xu, Liu, and Sun (2012), Zeng, Li, Liu, Zhang, and Chen (2015), all of which mainly incorporate the use of equivalence and similarity relations. The notion of R-fuzzy was the first approach that used rough sets to approximate the membership functions of fuzzy sets (Yang & Hinde, 2010).

Section 2 will go on to provide the foundational preliminaries for fuzzy sets, rough sets and R-fuzzy sets, along with their associated notations. A worked example involving human perception regarding noise pollution using an R-fuzzy approach is demonstrated. Section 3 introduces the newly derived significance degree. The noise pollution example is further extended by the implementation of the significance measure, to quantify the meaning and intent of the encapsulated membership values. In addition, a human perception based example regarding visualisation is also put forward. Section 4 describes the relationships that exist between R-fuzzy sets, the significance measure and traditional fuzzy sets. The equivalence between the coupling of an R-fuzzy set and the significance measure, to that of a type-2 fuzzy set is remarked upon. Section 5 provides the reader with remarks, where the strengths and weaknesses of the proposed research are discussed, along with theoretical comparisons to other approaches. Section 6 draws out the conclusion and summarises upon the advantages of using an R-fuzzy approach in conjunction with a significance degree measure for human perception based modelling. Final remarks, possible enhancements for future work are also discussed.

1.1. Motivation

The motivation for this paper comes from the desire to extend the applicability of R-fuzzy sets. As a result, the novelty of this paper is with regards to providing a means to quantify the importance of each membership value contained within an R-fuzzy set. The newly derived significance measure can also act as a validator for values contained within the lower approximation, as the returned value should be an absolute 1. Equally, if the membership values were completely disregarded the returned value would be an absolute 0, as they would not be included within the rough set. Any returned value within the interval [0, 1] signifies that the membership value has some importance to some degree. This echoes the sentiment of a typical type-1 fuzzy set and in doing so, enhances the overall existing robustness and versatility of R-fuzzy sets, increasing its scope for applicability. The better understood a problem, the more better equipped the solution.

According to Klir and Wierman (1998) there exist three kinds of general uncertainty. Real world problems often involve uncertainty, from an empirical level, uncertainty is often associated with any type of measurement. Resolution can be a cause for concern when involving exactness; 0.1 is different from 0.01 as it to 0.001, and so on. From the cognitive level, uncertainty exists in the vagueness and ambiguity associated with natural language. Your understanding of a word may not have to be exact match to the person you are conversing with, suffice to say, an overlap of an understanding can still act as an agreement of the sentiment nonetheless. At the social level, uncertainty can be used to ones advantage, where it is often simulated by individuals for different purposes; privacy, secrecy and propriety (Klir & Wierman, 1998).

There could be several root causes for the existence of uncertainty. The information associated to the problem may be inherently noisy or incomplete, riddled with contradictions, vague and ambiguous. These deficiencies may result in sub-faceted aspects of uncertainty, uncertainty within uncertainty.

Therefore the categorised three states of uncertainty are given as vagueness, associated to fuzzy with respect to imprecise, vague boundaries of fuzzy sets. Imprecision, this is with regards to nonspecificty of the cardinalities of sets and their alternatives. Finally, discord, with regards to strife which expresses conflicts and contradictions of the various sets of alternatives (Klir & Folger, 1988; Klir & Wierman, 1998).

Klir and Wierman (1998) then go onto divide the aforementioned three main types of uncertainness into two distinct classes, fuzziness and ambiguity. These remarks are also echoed by Berenji (1988). The need for higher dimensionality for uncertainty encapsulation makes a generalised type-2 fuzzy logic approach very appealing. If one could lessen the burden of complexities of a type-2 approach, it would allow for a greater scope of applicability. The