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Nash decomposition for process efficiency in multistage production systems

M.A. Hinojosa[a,*], S. Lozano[b], A.M. Mármol[c]

[a] Departamento de Economía, Métodos Cuantitativos e Historia Económica, Universidad Pablo de Olavide, Ctra. de Utrera Km. 1, Sevilla 41013, Spain
[b] Departamento de Organización Industrial y Gestión de Empresas I, Universidad de Sevilla, Spain
[c] Departamento de Economía Aplicada III, Universidad de Sevilla, Spain

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ABSTRACT

Many production systems consist of a sequence of processes or stages. For these systems, relational network DEA can be used and an overall system efficiency (equal to the efficiency of the different processes) can be computed. However, there can be alternative solutions that give different estimations of the process efficiencies and therefore lead to different decompositions of the overall system efficiency. It is not obvious which efficiency decomposition to use. In this paper, it is shown how a Nash bargaining game can be used to compute point estimates of the efficiency of the processes for multistage systems. The proposed approach extends and improves over existing approaches for production systems with just two stages. The rationality principles behind the proposed solution approach are presented and an interesting interpretation of the resulting efficiency decomposition is provided. The fact that this rigorous solution approach leads to such a simple and elegant efficiency decomposition should facilitate its adoption by Expert and Intelligent Systems practitioners.

1. Introduction

Frontier analysis methods aim at benchmarking a set of operating units (commonly termed Decision Making Units, DMU) against each other so as to identify the best performers (i.e. those that exhibit the best practices), uncovering and measuring existing inefficiencies in the production process. Among frontier analysis methods one of the most often used techniques is Data Envelopment Analysis (DEA), which is a deterministic, Linear Programming-based, non-parametric tool for assessing the relative efficiency of the units to be benchmarked.

Conventional DEA models consider a DMU as a black box that directly transforms inputs into outputs. There exist however a number of DEA applications in which several interrelated stages are distinguished so that intermediate products, internally generated and consumed within the system, are also considered. This more fine-grained approach is generally labeled Network DEA (e.g. Färe & Grosskopf, 1996, 2000). Many Network DEA models have been proposed in the last years, including, among others, game-theory approaches (Liang, Cook, & Zhu, 2008), relational Network DEA (Kao & Hwang, 2008, 2010), weighted additive efficiency decomposition (Chen, Cook, Li, & Zhu, 2009; Cook, Zhu, Bi, & Yang, 2010), Network Slack-Based Measure (NSBM) of efficiency (Lozano, 2015b; Tone & Tsutsui, 2009), Slacks-Based Inefficiency measure (Fukuyama & Weber, 2010), scale and cost network DEA efficiency (Lozano, 2011), dynamic Network DEA (Tone & Tsutsui, 2014), Malmquist index approach (Kao & Hwang, 2014), etc. Kao (2014b) and Halkos, Tzeremes, and Kourtzidis (2014) provide extensive and up-to-date reviews of Network DEA models. As regards network DEA applications, the list is also long and includes many sectors like banking (e.g. Wanke & Barros, 2014; Kwon & Lee, 2015; Lozano, 2015c), environmental performance (e.g. Lozano, 2015a), transportation (e.g. Lozano & Gutirrez, 2014), sports (e.g. Moreno & Lozano, 2014), etc.

Some of the proposed Network DEA models are based on the so-called multiplier DEA formulation. These models may have multiple alternative optima and therefore alternative efficiency decompositions are possible (see, e.g. Kao & Hwang, 2008; Liang et al., 2008). This problem can also affect the decomposition into technical and scale efficiencies (Kao & Hwang, 2011) and not only in the case of two-stage systems, but also in the case of the efficiency decomposition of general multistage systems (Kao, 2014a). In this type of production systems in series the overall system efficiency is the product of the efficiency of the different processes. Expressing the overall system efficiency as the product of the process efficiency is called efficiency decomposition. The problem is

* Corresponding author. Tel.: +34 618525509.
E-mail addresses: mahinram@upo.es (M.A. Hinojosa), slozano@us.es (S. Lozano), amarmol@us.es (A.M. Mármol).

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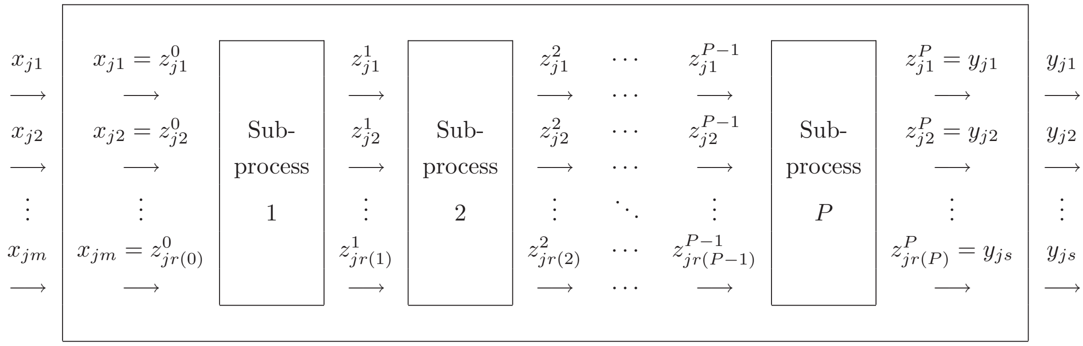

Fig. 1. Production process of DMU j ($j = 1, 2, \dots, n$).

that there can be different process efficiency scores that correspond to the same level of overall efficiency. This means that there can be multiple alternative efficiency decompositions and it is not obvious which efficiency decomposition to use.

In the case of two-stage systems, some authors have proposed different ways of solving the uncertainty about how the processes efficiencies should be computed. One approach is to compute the best and worst possible efficiency scores of each process, by choosing the best score for one process and the worst for the other process, depending on which process efficiency is the decision maker more concerned with (see, e.g., Liang et al., 2008; Kao & Hwang, 2014). The problem with this approach is that the analyst has to establish an order or ranking of the importance of the processes, something which is neither easy nor practical in the case of more than two stages. Recently, Despotis, Sotiros, and Koronakos (2014, 2015) have approached the efficiency decomposition problem from a multiobjective perspective. They propose a model for computing an Ideal Point with the largest possible efficiency scores of each stage and then use the lexicographic weighted Chebycheff method to determine the process efficiencies. An important feature of this approach is that it can be applied to general multistage networks.

Another alternative also proposed in the case of two-stage systems is to look for efficiency decompositions based on game theory and which can be regarded as "fair" in relation to some rationality principles. Thus, Liang et al. (2008) proposed a Stackelberg game. This type of leader-follower game is difficult to extend also in the case of multistage systems and, again, would imply an ordering of the importance of the different processes.

Following a different venue, but also for the case of two-stage systems, Du, Liang, Chen, Cook, and Zhu (2011) and Zhou, Sun, Yang, Liu, and Ma (2013) have proposed the use of the Nash bargaining solution (Nash, 1950). This is a cooperative game approach and, in principle, can be extended to the multistage systems. This is what our paper aims at, but in order to compute a solution for the efficiency decomposition and to identify the properties on which it is supported, the so called Nash extension solution (Conley & Wilkie, 1996) is considered. This is so because, as it is shown in the paper, the efficiency decomposition feasibility region is not convex and therefore the conventional Nash solution used in Du et al. (2011) and Zhou et al. (2013) is not appropriate. We prove, however, that for this class of problems, the decomposition generated by the Nash extension solution coincides with that obtained by applying the Nash solution and can be computed by using a simple and elegant formula. We also provide the interpretation of the properties that the solution fulfills in this context. Summarizing this point, there is not currently any approach for efficiency decomposition of multistage production systems with more than two stages and of those methods that have been proposed for two-stage systems some are not workable for multiple stages and others can be extended but with caution and with the appropriate adjustments as discussed in this paper.

The structure of the rest of the paper is the following. In Section 2 the Network DEA models to compute the overall efficiency of the system, as well as those which permit to determine the best and worst process efficiencies are presented. Section 3, includes a brief review of the bargaining theory needed to develop our results. In Section 4, the proposed approach is presented and discussed. Section 5 contains numerical examples to illustrate the approach, while Section 6summarizes and concludes.

2. Efficiency decomposition in a multistage system

Consider a DEA model in which each DMU is a series production system organized internally as a sequence of P processes or stages. For each DMU j, $j = 1, 2, \dots, n$,

(i) $\mathbf{x}_j = (x_{j1}, x_{j2}, \dots, x_{jm})^t$ represents the vector of m exogenous inputs, which are the inputs of the first sub-process,
(ii) for each sub-process $p = 1, 2, \dots, P-1$, $\mathbf{z}^p_j = (z^p_{j1}, z^p_{j2}, \dots, z^p_{jr(p)})^t$ represents the output vector from stage p, which is the only input vector to the stage $p+1$,
and
(iii) $\mathbf{y}_j = (y_{j1}, y_{j2}, \dots, y_{js})^t$ represents the vector of the s outputs resulting from the last sub-process, P, which are the final outputs corresponding to DMU j.

For each $j = 1, 2, \dots, n$, the exogenous inputs, $\mathbf{x}_j$, and the final outputs, $\mathbf{y}_j$, are denoted respectively by $\mathbf{z}^0_j$ and $\mathbf{z}^P_j$. Denote also the number of exogenous inputs, m, and the number of final outputs, s, of each DMU by $r(0)$ and $r(P)$, respectively. The production process of each DMU, j ($j = 1, 2, \dots, n$), can be represented as shown in the Fig. 1:

The efficiency score of a specific DMU, $J \in \{1, 2, \dots, n\}$, in the multistage system can be computed by solving the following optimization problem:

$$E(J) = \max \frac{\sum_{r=1}^{r(P)} w^P_{Jr} z^P_{Jr}}{\sum_{r=1}^{r(0)} w^0_{Jr} z^0_{Jr}}$$

$$s.t.\quad \frac{\sum_{r=1}^{r(p)} w^p_{Jr} z^p_{jr}}{\sum_{r=1}^{r(p-1)} w^{p-1}_{Jr} z^{p-1}_{jr}} \le 1, \quad \forall\, j = 1, 2, \dots, n, \quad \forall\, p = 1, \dots, P;$$

$$w^p_{Jr} \ge 0, \quad \forall\, p = 0, 1, \dots, P, \quad \forall\, r = 1, 2, \dots, r(p). \tag{1}$$

where, for each $p = 0, 1, \dots, P$, w^p_J are the vectors of weights for the exogenous inputs ($w^0_J \in \mathbb{R}^{r(0)}_+$), intermediate products ($w^p_J \in \mathbb{R}^{r(p)}_+$, $p = 1, 2, \dots, P-1$), and final outputs ($w^P_J \in \mathbb{R}^{r(P)}_+$), respectively.

By using Charnes and Cooper's (1962) transformation, and maintaining the notation z^p_{jr} to represent the variables in the new