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# A new unconstrained global optimization method based on clustering and parabolic approximation

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ABSTRACT

A new unconstrained global optimization method based on clustering and parabolic approximation (GOBC-PA) is proposed. Although the proposed method is basically similar to other evolutionary and stochastic methods, it represents a significant advancement of global optimization technology for four important reasons. First, it is orders of magnitude faster than existing optimization methods for global optimization of unconstrained problems. Second, it has significantly better repeatability, numerical stability, and robustness than current methods in dealing with high dimensionally and many local minima functions. Third, it can easily and faster find the local minimums using the parabolic approximation instead of gradient descent or crossover operations. Fourth, it can easily adapted to any theoretical or industrial systems which are using the heuristic methods as an intelligent system, such as neural network and neuro-fuzzy inference system training, packing or allocation of objects, game optimization problems. In this study, we assume that the best cluster center gives the position of the possible global optimum. The usage of clustering and curve fitting techniques brings multi-start and local search properties to the proposed method. The experimental studies, such as performed on benchmark functions, a real world optimization problem and tuning the neural network parameters for classification problems, show that the proposed methodology is simple, faster and, it demonstrates a superior performance when compared with some state of the art methods.



## 1. Introduction

Global optimization methods have been used in many scientific areas such as engineering, financial, medical, military, etc. Nowadays, determining the global optima of nonlinear, non-differentiable and non-convex functions is still an unsolved problem.

A global optimization problem can be formalized as a pair($\mathbf{X}$, $E$), where $\mathbf{X} \subseteq \Re^n$ is a bounded input set on $R^n$ and $E$: $\mathbf{X} \to \Re$ is an $n$-dimensional real-valued objective function. The problem is to find a point $\mathbf{X}_* \in \mathbf{X}$ such that $E(\mathbf{X}_*)$ is the global optimum on $\mathbf{X}$ (Yao, Liu, & Lin, 1999).

Nowadays, global optimization methods are classified as deterministic, stochastic and heuristic methods. These methods can be called computational intelligence algorithms. The most popular computational intelligence algorithms are particle swarm optimization (Kennedy & Eberhart, 1995), genetic algorithm (Goldberg, 1989), simulated annealing (Kirkpatrick, Gelatt, & Vecchi, 1983), ant colony optimization (Dorigo, Maniezzo, & Colorni, 1996), tabu search (Glover, 1989), differential evolution (Price, Storn, & Lampinen, 2005). The heuristic methods enclose the evolutionary methods, which are simple modeling of natural events. Especially, swarm based methods such as particle swarm, artificial bee (Karaboga & Akay, 2009), ant colony, differential evolution, bird flocking (Antoniou, Pitsillides, Blackwell, & Engelbrecht, 2009), bacterial foraging (Passino, 2002), and so on are very popular and effective methods for global optimization.

In global optimization problems, the major challenge is that an algorithm may be trapped to the local optimum of the objective function. In evolutionary methods, some of the genetic operations, such as mutation, crossover, and randomly regeneration are used to avoid local optima. In swarm based methods, the particle movements are adapted with best solutions. Clustering methods can also be used as local search methods to detect the local optima or to partition the solution space.

Global optimization methods based on clustering can be viewed as a modified form of the standard multi-start procedure, which

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performs a local search from several points distributed over the entire search domain (Liu & Tian, 2015). A drawback of multi-start is that when many starting points are used the same local minimum may be identified several time, thereby it leads to an inefficient global search. Clustering methods attempt to avoid this inefficiency by carefully selecting points at which the local search is initiated. If this procedure successfully identifies the groups that represent the neighborhoods of local minima, then redundant local searches can be avoided by simply starting a local search for any point within each cluster. Kan and Timmer (1987) proposed multi-level single linkage (MLSL) method based on clustering for global optimization. The MLSL has multi-start and local search properties. Flores (2010) investigated the effectiveness of the usage of clustering techniques and stopping conditions for global optimization in the particular case of robust regression. Wang, Zhang, and Zhang (2007) used hierarchical clustering algorithm to avoid redundant search in differential evolution (DE) algorithm. By this means, they accelerated the traditional DE. Cai, Gong, Ling, and Zhang (2011) used one-step $K$-means instead of multi-parent crossover operators in DE algorithm. Their method resulted in better performance than Wang's method for the same problems. Liu, Li, Nie, and Zheng (2012) improved the DE algorithm using a one-step $K$-means clustering and two multi-parent crossovers. Their objectives were to establish the population to keep diversity at early stages to explore the most promising regions. Han, Lin, Chang, and Li (2013) proposed a group-based differential evolution (GDE) algorithm for numerical optimization problems. Initially, all individuals in the population were partitioned into an elite group and an inferior group based on their fitness value. Subsequently, the GDE algorithm employed crossover and selection operations to produce offspring for the next generation. Halder, Das, and Maity (2013) proposed a cluster-based dynamic differential evolution with external archive for global optimization in dynamic fitness landscape. Their method used a multi population method where the entire population was partitioned into several clusters according to the spatial locations of the trial solutions. Then the clusters were evolved separately using a standard differential evolution algorithm. Gao, Yen, and Liu (2014) used a cluster-based differential evolution (DE) for multimodal optimization problems. The clustering partition was used to divide the whole population into subpopulations. Then, the self-adaptive parameter control was employed to enhance the search ability of DE. Their multi-population strategy and the self-adaptive parameter control technique were applied to two versions of DE, crowding DE (CDE) and species-based DE (SDE).

Zhu, Liu, Long, and Zhao (2012) proposed a heuristic global optimization method using adaptive radial basis function based on fuzzy clustering. The fuzzy $c$-means clustering method was employed to identify the reduced attractive regions in the original design space. Wang and Simpson (2004) proposed an intuitive methodology to systematically reduce the design space to a relatively small area. The attractive regions were defined by fuzzy $c$-means clustering. Chen and Liao (2013) presented an efficient cluster-based tribes optimization algorithm (CTOA) for designing a functional-link-based neuro-fuzzy inference system (FLNIS) for production applications. They used CTOA for optimizing the parameters of the FLNIS model. They split up the swarm into multiple tribes using self-clustering algorithm, and then used different translation strategies to update each particle.

Kennedy (2000) improved the particle swarm optimization (PSO) performance with $K$-means clustering. Li and Yang (2009) proposed a particle swarm optimization method for non-stationary optimization problems. Their dynamic optimization method was also used in the single linkage hierarchical clustering to track multiple peaks based on the nearest neighbor search strategy. Liang, Li, Zhang, and Zhou (2015) proposed adaptive PSO based on clustering (APSO-C), by considering the population topology and individual behavior control together to balance local and global search in an optimization process. APSO-C used $K$-means clustering operation for dividing the swarms.

Zhao and Wang (2016) proposed a stochastic optimization algorithm inspired by a behavior of bacteria. This algorithm determines the direction of random searches chemotactic mimic movements. Wu, Mallipeddi, Suganthan, Wang, and Chen (2016) proposed a method which has multi population and consisting of multiple strategies. They benefited from differential evolution (DE). Cui, Li, Lin, Chen, and Lu (2016) used differential evolution (DE) with sub-populations (MPADE) in their study. Parent population divided three sub-population based on the fitness values in their algorithms. When they compare MPADE with DE methods, they saw MPADE is much better than DE. Li, Zhao, Weng, and Han (2016) developed a new swarm intelligence algorithm. This new nature-inspired algorithm which is named cognitive behavior optimization (COA) consists of three stages: exploration, communication and adjustment. Meng, Li, Yin, Chen, and Guo (2016) proposed a novel hybrid optimization algorithm. They have developed to eliminate PSO's some disadvantages. They used crisscross search optimization (CSO). They were inspired genetic algorithm (crossover operation) and by the Confucian doctrine of the golden mean.

If we look at the new papers in the literature preview, we will see that the mutation strategies and the search mechanism are trying to improve. While Cui et al. (2016); Meng et al (2016) and Wu et al. (2016) improving the crossover operation, Zhao and Wang (2016) and Li et al. (2016) trying for better search mechanism. The differences between our approach and the others are; there isn't any mutation step and gradient descent methods in the GOBC-PA. In addition to that, GOBC-PA use parabolic approximation and clustering for search mechanism to shorten the convergence time. These features make GOBC-PA is a novel, faster and better performance in the literature survey.

In general, clustering algorithms are applied to populations for narrowing the areas of possible searching, and speeding the algorithms. In this study, we proposed a new multi-start global optimization method based on clustering and parabolic approximation (GOBC-PA), which does not use traditional mutation or crossover operation. The crossover is partially applied to the selected cluster centers and the best individuals in this study. The method generates some individuals of next generation according to the clustering results of the previous generation. In GOBC-PA, the created populations are clustered according to their evaluation values. The similar and the closest points can be clustered into the same group as in the social neighborhood strategy. The cluster centers are evaluated by their objective function values, and then sorted. The best cluster centers denote the possible optimal regions and search directions. The complex optimization problems can also be partitioned into the narrowed regions with clusters, and some of the cluster regions have convex or concave forms. If these regions are fitted with the second order polynomials (parabolas), the vertices of parabolas can approximate the local maxima or minima of the optimization problem. The coefficients of parabolas are easily determined by the quadratic least squares estimation (LSE) method. When the vertex coordinates of parabola are closer to local optimum than the cluster center coordinates, the cluster center coordinates are changed with the vertex coordinates of parabola. The next generation contains the best cluster centers, the best of two evaluation points, and randomly generated three points which are around the each best point. The residual individuals of the new generation are randomly created instead of eliminating the worst points from the working space $\mathbf{X}$ to avoid the undesirable local optimums. With this approach, the traditional mutation and crossover operations are eliminated from GOBC-PA method. The partial crossover is only used to produce new individuals that are neighbor of the selected cluster centers