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Reliability analysis of the complex mode indicator function and Hilbert Transform techniques for operational modal analysis

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ABSTRACT

This study presents application of the CMIF and the Hilbert Transform techniques onto simulated response data obtained using a numerical model of a typical school building from Turkey. White noise is added to the data in order to achieve a noise to signal ratio of 5%. 100 Monte Carlo analysis sequences are carried out and the modal parameters (the frequencies, the mode shapes and the damping ratios) are identified at each Monte Carlo run for both techniques. The results are compared with the identifications obtained from the simulated data using stochastic subspace based system identification technique. The overall results of the study show that the mode shapes are clearly identified the best by using the CMIF technique. The damping ratios are estimated better by using the stochastic subspace based system identification technique whereas the frequencies are best determined by the CMIF. The results also show that both the CMIF and the Hilbert Transform techniques are sensitive to the type of window used as well as the averaging and the decimation process. It is apparent that the CMIF technique is as robust as the frequently used stochastic subspace based system identification technique and can be confidently used for modal parameter estimation of stiff low to mid rise reinforced concrete structures.

1. Introduction

Output only modal identification has been very popular in the last decade within the research circles due to its proven superiority regarding the ease of application and lower expense compared to the input output modal identification. The output only modal identification is more suited to large civil engineering structures which are difficult to excite to appreciable amplitudes by artificial excitation sources. Within this context, two alternative approaches are present in the literature: namely, the time domain techniques (Brincker, Zhang, & Andersen, 2000) and the frequency domain techniques (Hermans & Van der Auweraer, 1999). The complex mode indicator function, which is a singular value decomposition enhancement of the classical peak picking technique has been used extensively in the literature (Peeters, 2000). Recently, a further enhancement to this technique has been recommended which applies the Hilbert Transform on the amplitude spectra achieved from the autocorrelation functions of the outputs at each sensor degrees of freedom (Agneni et al., 2003; Agneni, Brincker, & Capotelli, 2004). Agneni, Balis Crema, and Coppotelli (2010) proposes a technique to evaluate the biased frequency response functions. In this study, the technique proposed by Agneni et al. (2010) is compared to the complex mode indicator function (CMIF) technique using 100 Monte Carlo simulations. Both techniques are applied on the numerically obtained output data from the assumed sensor locations of a stiff reinforced concrete building. This building is a typical school building located in the North Western part of Turkey and is currently instrumented with 17 sensors and a data acquisition system with a high dynamic range within the context of a TUBITAK project (Gundes Bakir, 2008). However, the experimentally obtained data is not used in this study. This is because, the aim of this study is to investigate the reliability bounds of the CMIF and the Hilbert Transform Techniques. Therefore, it is imperative to know beforehand the real modal parameters (frequencies, mode shapes and the damping ratios) in order to compare them with those obtained from the two frequency domain system identification techniques, namely, the CMIF and the Hilbert Transform. The building is thus modeled with finite element (FE) model and is subjected to white noise excitation. The output data obtained this way is further modified by adding white noise so that the noise to signal ratio is 5%. The modal parameter estimates obtained from both techniques are compared to the real modal parameters obtained from the FE model. The results show that the CMIF technique gives better estimates for the mode shapes and the eigenfrequencies as compared to the stochastic subspace based system identification technique. It is also apparent that the Hilbert Transform Technique does not substantially improve the modal parameters obtained from the CMIF technique.

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2. Complex mode indicator function

Detailed overviews for the CMIF method are provided in Brincker et al. (2000), Allemang and Brown (2006), Shih, Tsuei, Allemang, and Brown (1988a, 1988b) and Shih (1989). In this section, the technique will be summarized for purposes of brevity. The power spectral density of the response can be defined as:

$$G_{yy}(j\omega) = H^*(j\omega)G_{uu}(j\omega)H(j\omega)^T \tag{1}$$

Here, $G_{uu}(j\omega)$ is the $m \times m$ power spectral density (PSD) matrix of the input, m is the number of inputs, $G_{yy}(\omega)$ is the $\ell \times \ell$ PSD matrix of the responses, $H(j\omega)$ is the $\ell \times m$ frequency response function and the $(\cdot)^*$ and $(\cdot)^T$ denote the complex conjugate and the transpose, respectively. If partial fraction expansion is applied on the frequency response function, the following expression is obtained:

$$H(j\omega) = \sum_{k=1}^{n} \frac{R_k}{j\omega - \lambda_k} + \frac{R_k^*}{j\omega - \lambda_k^*} \tag{2}$$

where n is the number of modes, λ_k is the pole and R_k is the residue that can be expressed as:

$$R_k = \Phi_k \Upsilon_k^T \tag{3}$$

where $\Phi_k = [\phi_{1k}, \phi_{2k}, \ldots, \phi_{Nk}]^T$ and $\Upsilon_k = [\gamma_{1k}, \gamma_{2k}, \ldots, \gamma_{N_{ref}k}]^T$ are the k^{th} mode shape vector and the modal participation vector, respectively. N_{ref} is the number of inputs or references. $H(j\omega)$ becomes a square matrix and $\Upsilon_k = \Phi_k$ if all outputs are used as references. Pole k, λ_k is defined as:

$$\lambda_k = -\sigma_k + i\omega_{dk} \tag{4}$$

where σ_k is the damping factor; ω_{dk} is the damped modal frequency. The modal frequency or the undamped natural frequency can be expressed as:

$$\omega_k = \sqrt{\sigma_{k+}^2 \omega_{dk}^2} \tag{5}$$

The damping ratio can be computed from:

$$\xi_k = \frac{\sigma_k}{\sqrt{\sigma_{k+}^2 \omega_{dk}^2}} \tag{6}$$

If these formulas are applied in output only modal analysis, the input has to be assumed as white noise. In this case, the power spectral matrix $G_{xx}(j\omega)$ becomes a constant matrix C, then the power spectral density matrix G_{yy} becomes:

$$G_{yy}(j\omega) = \sum_{k=1}^{n}\sum_{s=1}^{n}\left[\frac{R_k}{j\omega - \lambda_k} + \frac{R_k^*}{j\omega - \lambda_k^*}\right] C \left[\frac{R_s}{j\omega - \lambda_s} + \frac{R_s^*}{j\omega - \lambda_s^*}\right]^H \tag{7}$$

where $(\cdot)^H$ shows the complex conjugate transpose or Hermitian operation. The output power spectral density can be reorganized in a pole/residue form after some mathematical manipulations using the Heaviside partial fraction theorem as,

$$G_{yy}(j\omega) = \sum_{k=1}^{n} \frac{A_k}{j\omega - \lambda_k} + \frac{A_k^*}{j\omega - \lambda_k^*} + \frac{A_k^H}{-j\omega - \lambda_k^*} + \frac{A_k^T}{-j\omega - \lambda_k} \tag{8}$$

where A_k is the corresponding residue matrix. The residue matrix is an $\ell \times \ell$ Hermitian matrix which can be expressed as,

$$A_k = R_k C \left(\sum_{s=1}^{n} \frac{R_s^H}{-\lambda_k - \lambda_s^*} + \frac{R_s^T}{-\lambda_k - \lambda_s} \right) \tag{9}$$

If the structure is lightly damped (i.e., $\sigma_k = \omega_{dk}$), the residue in the vicinity of the kth modal frequency can be derived approximately by using Eq. (3) as,

$$A_k \approx \frac{R_k C R_k^H}{2\sigma_k} = q_k \Phi_k \Phi_k^H \tag{10}$$

where the scalar q_k can be expressed as,

$$q_k = \frac{\gamma_k^T C \gamma_k^*}{2\sigma_k} \tag{11}$$

At a certain frequency ω, only a limited number of modes will contribute significantly (Brincker, Zhang, & Andersen, 2001). If this subset of modes is denoted by $Sub(\omega)$, the response spectral density can be expressed as,

$$G_{yy}(j\omega) = \sum_{k \in Sub(\omega)} \frac{q_k \phi_k \phi_k^T}{j\omega - \lambda_k} + \frac{q_k^* \phi_k^* \phi_k^{T^*}}{j\omega - \lambda_k^*}. \tag{12}$$

The singular value decomposition (SVD) is the tool that is used to compute the rank of a matrix where the number of nonzero singular values equals the rank (Golub & Van Loan, 1996). The technique is applied on the spectrum estimate $G_{yy} \in C^{\ell \times \ell}$ which can be expressed as the discrete time Fourier transform of the covariance sequence R_k as,

$$G_{yy}(e^{j\omega\Delta t}) = \sum_{k=-\infty}^{\infty} R_k(e^{-j\omega k \Delta t}) \tag{13}$$

Reference sensors can be selected and used in the computation of the spectrum estimate (Peeters, 2000). The reference outputs can be selected from the full output matrix as,

$$y_k^{ref} = L y_k, \quad L = (I_r 0) \tag{14}$$

where $y_k^{ref} \in \mathbb{R}^r$ are the reference outputs and $L \in \mathbb{R}^{r \times \ell}$ is the selection matrix that selects the reference sensors. In this case, the reduced spectrum matrix $G_{yy}^{ref} \in C^{\ell \times r}$ can be obtained as,

$$G_{yy}^{ref} = G_{yy} L^T \tag{15}$$

The weighted averaged periodogram technique is used for estimating the spectra. First, the Discrete Fourier Transform (DFT) of the weighted output signal is calculated as,

$$Y(e^{j\omega\Delta t}) = \sum_{k=0}^{N-1} w_k y_k e^{-j\omega k \Delta t} \tag{16}$$

where w_k represents the window function used. The spectrum is then approximated by multiplying the DFT of the output by its complex conjugate transpose and scaling this product by the squared norm of the window as,

$$\widehat{G}_{yy}(e^{j\omega\Delta t}) = \frac{1}{\sum_{k=0}^{N-1} |w_k|^2} Y(e^{j\omega\Delta t}) Y^T(e^{-j\omega\Delta t}) \tag{17}$$

Subsequently, the SVD of the spectrum matrix is obtained as,

$$G_{yy}(s) = U(s)\Sigma(s)U^H(s) \tag{18}$$

where U and V are the orthogonal matrices formed by the N singular vectors u^N and v^N, i.e., $U^H U = I$ and $V^H V = I$ and $U(s) = [u_{i1}, u_{i2}, \ldots, u_{ik}]$ is a complex unitary matrix that holds the singular vectors u_{ij}. The matrix $\Sigma(s)$ is composed of the singular values in descending order in its diagonal and its plot versus the frequency gives the actual CMIF diagram. The maxima in the CMIF plot gives the eigenfrequencies. If only one mode is contributing at a certain frequency, there will be only one term in Eq. (12). Then the first singular vector is an estimate of the mode shape.

$$\hat{\phi} = u_{i1} \tag{19}$$

The corresponding singular value is the autopower spectral density function of the corresponding SDOF system as shown in Eq. (12). From this piece of the single degree of freedom (SDOF) density function obtained around the peak of the G_{yy}, the natural frequency and the damping can be identified using SDOF techniques as extensively covered in the books of Maia et al. (1997), Allemang (1999), Ewins