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A branch-and-cut approach for the vehicle routing problem with loading constraints

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ABSTRACT

In this paper we describe a branch-and-cut algorithm for the vehicle routing problem with unloading constraints. The problem is to determine a set of routes with minimum total cost, each route leaving a depot, such that all clients are visited exactly once. Each client has a demand, given by a set of items, that are initially stored in a depot. We consider the versions of the problem with two and tri dimensional parallelepiped items. For each route in a solution, we also need to construct a feasible packing for all the items of the clients in this route. As it would be too expensive to rearrange the vehicle cargo when removing the items of a client, it is important to perform this task without moving the other client items. Such packings are said to satisfy unloading constraints.

In this paper we describe a branch-and-cut algorithm that uses several techniques to prune the branch-and-cut enumeration tree. The presented algorithm uses several packing routines with different algorithmic approaches, such as branch-and-bound, constraint programming and metaheuristics. The careful combination of these routines showed that the presented algorithm is competitive, and could obtain optimum solutions within significantly smaller computational times for most of the instances presented in the literature.

1. Introduction

Several problems in transportation systems involve route planning for vehicles with containers attached and accommodation of cargo into these containers. The route planning problem and the packing problem are well known problems in the research literature, and they were largely explored separately. However, in recent years there has been some interest in considering both problems combined, leading to better global solutions.

In the Vehicle Routing Problem with D-Dimensional Unloading Constraints (DL-CVRP), clients have a demand for goods stored into a depot, represented by D-dimensional parallelepipeds, and k vehicles must be used to deliver these goods. Each travel from the depot to a client, or from a client to a next one has a cost. The problem is to find k routes leaving the depot, one route for each vehicle, such that all clients are visited exactly once, and such that the items of clients of a route can be packed in the vehicle's container. The objective function of the problem is to minimize the total cost of the routes. As it would be too expensive to rearrange the cargo at each visit, we add a condition that the goods to be unloaded in a client must be removed without moving the remaining goods, these are the so called *unloading constraints*. We consider two versions of the problem, one with two dimensional items and another with three-dimensional ones.

The literature in vehicle routing problem is extensive, with different variants including practical constraints. In the last decade, there has been some interest in variants that include two- and three-dimensional packing constraints. Iori, Salazar-González, and Vigo (2007) were the first to present an exact branch-and-cut approach for the capacitated vehicle routing problem with two-dimensional unloading constraints (2L-CVRP). To separate infeasible routes, these authors used known separations routines for the CVRP. To separate routes that lead to infeasible packings, they used an adaptation of the exact algorithm, presented in Martello, Pisinger, and Vigo (2000), to satisfy unloading constraints. Following this work, Azevedo, Hokama, Miyazawa, and Xavier (2009) also presented an exact method for 2L-CVRP, using a different set of separation routines for the CVRP, made available by Lysgaard, Lechtford, and Eglese (2003). To cut routes that lead to infeasible packings, these authors also used an adaptation of the algorithm presented by Martello et al. (2000). Due to the difficulty of solving this problem exactly, several heuristics were also proposed. Gendreau, Iori, Laporte, and Martello (2008) presented a tabu search method to the 2L-CVRP. Fuellerer, Doernera, Hartla, and Iori (2009) employed an ant colony method. Zachariadis, Tarantilis, and Kiranoudis (2009) introduced a guided tabu search method.

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Duhamel, Lacomme, Quilliot, and Toussaint (2011) presented a GRASP approach for the case without unloading constraints. Silveira and Xavier (2014) considered the pick-up and delivery version of the problem. In this case both loading and unloading constraints must be taken into account when generating a route. They presented an exact algorithm and also a GRASP heuristic for the problem. Approximation algorithms for the associated packing problem that considers unloading constraints were proposed by Silveira, Miyazawa, and Xavier (2013a, 2014); Silveira, Xavier, and Miyazawa (2013b).

The heterogeneous fleet variant with unloading constraint , where vehicles containers have different sizes, was considered by Wei, Zhang, and Lim (2014), who present an adaptive variable neighbourhood search metaheuristic for the three dimensional case. Dominguez, Juan, Barrios, Faulin, and Agustin (2014) presented a randomised multi-start biased metaheuristic for the two dimensional case.

The capacitated vehicle routing problem with three-dimensional unloading constraints (3L-CVRP) was first considered by Gendreau, Iori, Laporte, and Martello (2006). They presented a tabu search method to solve the problem. Junqueira, Oliveira, Carravilla, and Morabito (2013) presented an exact model for the 3L-CVRP via an integer linear programming model with practical constraints, namely stability, multidrop and load-bearing strength. The formulation was able to solve instances of moderate size. The constraints considered differ from the ones here. For more references on routing and loading problems we refer to a survey by Iori and Martello (2013).

In the 2L-CVRP and 3L-CVRP problems we face a packing subproblem of determining if a set of items of one route can be packed in a bin. This is the so called Orthogonal Packing Problem (OPP). In this problem it is given a set of items and a bin with bounded dimensions, and the objective is to find a placement of these items in the bin, or prove that it is unfeasible. When the items and the bins are two-dimensional objects (resp., three-dimensional), we have the two-dimensional orthogonal packing problem - 2OPP (resp. the three-dimensional orthogonal packing problem - 3OPP). Clautiaux, Jouglet, Carlier, and Moukrim (2008) presented an efficient method for solving the 2OPP using Constraint Programming. Their work was further extended to include new bounds by Mesyagutov, Scheithauer, and Belov (2012a), and to consider the three-dimensional case by Mesyagutov, Scheithauer, and Belov (2012b). Recently the problem was considered with the unloading constraint by Côté, Gendreau, and Potvin (2014), who presented an exact algorithm using branch-and-cut and a set of lower bounds.

The Constraint Programming (CP) paradigm has been proved to be a very efficient method for solving many different problems (Rossi, Beek, & Walsh, 2006). This paradigm was well known by other areas but just in the last decades was rediscovered by researchers in the combinatorial optimization community. The Integer Linear Programming (ILP) is probably the most used method for solving combinatorial optimization problems (Wolsey, 1998). The use of both, CP and ILP together is a hot topic and has obtained good results for many problems (Hooker, 2011).

In this paper we propose an exact branch-and-cut algorithm to solve the 2L-CVRP and the 3L-CVRP, following the branch-and-cut approach presented in Azevedo et al. (2009); Iori et al. (2007), using more elaborate cutting plane routines to cut not only routes whose items cannot be packed in one bin, using CVRP routines (Lysgaard et al., 2003), but also fast routines that check the feasibility of packings for sub-routes. These cuts are obtained using algorithms to solve the OPP. To this purpose, we tested a number of original and adapted algorithms, and also sophisticated lower bounds that can prove that a set of items cannot be packed in a bin. Besides the adaptation of the exact packing algorithm in Martello et al. (2000), we also adapted and improved the constraint programming algorithm presented by Clautiaux et al. (2008). We also present new heuristics, one is based on the Bottom-Left heuristic, and another one is a BRKGA metaheuristic.

The declarative approach of the constraint programming technique allowed us to obtain a packing algorithm for which practical constraints are easily to be incorporated, as opposed to the branch and bound approach used in Iori et al. (2007), such as balancing constraints and weight distribution (Bischoff & Ratcliff, 1995), grouping items (Bischoff & Ratcliff, 1995) and more specific application constraints. The efficiency of the proposed algorithms are evaluated by an experimental analysis with instances from the literature, comparing them with other algorithms from the literature. The careful application of separation routines, not only made possible to obtain solutions for larger instances, but also to deal with the three-dimensional problem version, which previous experimental results showed difficulty to solve.

The proposed use of constraint programming with integer linear programming models, can be applied to other practical variants, as the multi-depot (Li, Pardalos, Sun, Pei, & Zhang, 2015) and cross-docking (Morais, Mateus, & Noronha, 2014) vehicle routing problems, among others.

2. Orthogonal packing problem with unloading constraints

In this section we formally describe the Orthogonal Packing Problem with Unloading Constraint (OPPUL). We present exact algorithms, some heuristics, and lower bounds for the two and three-dimensional version of this problem.

2.1. Problem description

The orthogonal packing problem with unloading constraint can be defined as follows: It is given a D-dimensional container B of dimensions $(W^1, \ldots, W^D)$ with total volume $\mathcal{V}(B) = \prod_{d=1}^{D} W^d$, where $W^d \in \mathbb{Z}^+$, $1 \leq d \leq D$; n sets of D-dimensional items $(I_1, \ldots, I_n)$, let $I = \bigcup_{v=1}^{n} I_v$. Each item $i \in I_v$ has dimensions $(w_i^1, \ldots, w_i^D)$, where $w_i^d \in \mathbb{Z}^+$. The volume of an item $i$is denoted by $\mathcal{V}(i) = \prod_{d=1}^{D} w_i^d$ and the volume of a set of items I is denoted by $\mathcal{V}(I) = \sum_{i \in I} \mathcal{V}(i)$. The problem is to find a *packing*$\mathcal{P}_I$ of the items I in the bin B that respects the *unloading constraints* in the direction of the last dimension D.

More precisely, a packing $\mathcal{P}_I$ of items I in a container $B = (W^1, \ldots, W^D)$ that satisfy unloading constraints is a function $\mathcal{P}_I : I \to [0, W^1) \times \ldots \times [0, W^D)$ such that:

(i) The packing must be orthogonal, i.e. the edges of the items must be parallel to the respective container's edges.

(ii) The packing must be oriented, i. e., the items must be packed in the original orientation given in I.

(iii) Items of I must be packed within the container's boundaries. That is, if the position where the item is packed is given by $\mathcal{P}_I(i) = (x_i^1, \ldots, x_i^D)$, for each $i \in I$, then

$$0 \leqslant x_i^d \leqslant x_i^d + w_i^d \leqslant W^d, \text{ for } 1 \leqslant d \leqslant D. \quad (1)$$

(iv) Items must not overlap. That is, if the region occupied by the item i is given by $\mathcal{R}(i) = [x_i^1, x_i^1 + w_i^1) \times \cdots \times [x_i^D, x_i^D + w_i^D)$ then

$$\mathcal{R}(i) \cap \mathcal{R}(j) = \emptyset, \quad \text{for all pairs } i \neq j \in I. \quad (2)$$

(v) Items belonging to a set I_v are not blocked by any item belonging to a set I_u if $u > v$. That is, if item $i$must be unloaded before item j, j cannot be packed in the region between i and the end of the container, in the unloading dimension D. More precisely consider the region that includes the item i and its way to the exit of the container defined by A packing $\mathcal{P}_I$ of $I = I_1 \cup \ldots \cup I_n$ in the bin B respects the unloading constraints if it satisfy