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Multi-objective grey wolf optimizer: A novel algorithm for multi-criterion optimization

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ABSTRACT

Due to the novelty of the Grey Wolf Optimizer (GWO), there is no study in the literature to design a multi-objective version of this algorithm. This paper proposes a Multi-Objective Grey Wolf Optimizer (MOGWO) in order to optimize problems with multiple objectives for the first time. A fixed-sized external archive is integrated to the GWO for saving and retrieving the Pareto optimal solutions. This archive is then employed to define the social hierarchy and simulate the hunting behavior of grey wolves in multi-objective search spaces. The proposed method is tested on 10 multi-objective benchmark problems and compared with two well-known meta-heuristics: Multi-Objective Evolutionary Algorithm Based on Decomposition (MOEA/D) and Multi-Objective Particle Swarm Optimization (MOPSO). The qualitative and quantitative results show that the proposed algorithm is able to provide very competitive results and outperforms other algorithms. Note that the source codes of MOGWO are publicly available at http://www.alimirjalili.com/GWO.html.

1. Introduction

There are different challenges in solving real engineering problems, which needs specific tools to handle them. One of the most important characteristics of real problems, which make them challenging, is multi-objectivity. A problem is called multi-objective if there is more than one objective to be optimized. Needless to say, a multiple objective optimizer should be employed in order to solve such problems. There are two approaches for handling multiple objectives: *a priori* versus *a posteriori* (Branke, Kaußler, & Schmeck, 2001; Marler & Arora, 2004).

The former class of optimizers combines the objectives of a multi-objective problem to a single-objective with a set of weights (provided by decision makers) that defines the importance of each objective and employs a single-objective optimizer to solve it. The unary-objective nature of the combined search spaces allows finding a single solution as the optimum. In contrary, *a posterior* method maintain the multi-objective formulation of multi-objective problems, allowing to explore the behavior of the problems across a range of design parameters and operating conditions compared to *a priori* approach (Deb, 2012). In this case, decision makers will eventually choose one of the obtained solutions based on their needs. There is also another type of handling multiple objectives called progressive method, in which decision makers' preferences about the objectives are considered during optimization (Branke & Deb, 2005).

In contrary to single-objective optimization, there is no single solution when considering multiple objectives as the goal of the optimization process. In this case, a set of solutions, which represents various trade-offs between the objectives, includes optimal solutions of a multi-objective problem (Coello, Lamont, & Van Veldhuisen, 2007). Before 1984, mathematical multi-objective optimization techniques were popular among researchers in different fields of study such as applied mathematics, operation research, and computer science. Since the majority of the conventional approaches (including deterministic methods) suffered from stagnation in local optima, however, such techniques were not applicable as there are not nowadays.

In 1984, a revolutionary idea was proposed by David Schaffer (Coello Coello, 2006). He introduced the concepts of multi-objective optimization using stochastic optimization techniques (including evolutionary and heuristic). Since then, surprisingly, a significant number of researches have been dedicated for developing and

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evaluating multi-objective evolutionary/heuristic algorithms. The advantages of stochastic optimization techniques such as gradient-free mechanism and local optima avoidance made them readily applicable to the real problems as well. Nowadays, the application of multi-objective optimization techniques can be found in different fields of studies: mechanical engineering (Kipouros et al., 2008), civil engineering (Luh & Chueh, 2004), chemistry (Gaspar-Cunha & Covas, 2004; Rangaiah, 2008), and other fields (Coello & Lamont, 2004).

Early year of multi-objective stochastic optimization saw conversion of different single-objective optimization techniques to multi-objective algorithms. Some of the most well-known stochastic optimization techniques proposed so far are as follows:

- Strength–Pareto Evolutionary Algorithm (SPEA) (Zitzler, 1999; Zitzler & Thiele, 1999).
- Non-dominated Sorting Genetic Algorithm (Srinivas & Deb, 1994)
- Non-dominated Sorting Genetic Algorithm version 2 (NSGA-II) (Deb, Pratap, Agarwal, & Meyarivan, 2002)
- Multi-Objective Particle Swarm Optimization (MOPSO) (Coello, Pulido, & Lechuga, 2004)
- Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D) (Zhang & Li, 2007)
- Pareto Archived Evolution Strategy (PAES) (Knowles & Corne, 2000)
- Pareto–frontier Differential Evolution (PDE) (Abbass, Sarker, & Newton, 2001).

The literature shows that these algorithms are able to effectively approximate the true Pareto optimal solutions of multi-objective problems. However, there is a theorem here called No Free Lunch (NFL) (Wolpert & Macready, 1997) that has been logically proved that there is no optimization technique for solving all optimization problems. According to this theorem, the superior performance of an optimizer on a class of problems cannot guarantee the similar performance on another class of problems. This theorem is the foundation of many works in the literature and allows researchers in this field to adapt the current techniques for new classes of problems or propose new optimization algorithms. This is the foundation and motivation of this work as well, in which we propose a novel multi-objective optimization algorithm called Multi-Objective Grey Wolf Optimizer (MOGWO) based on the recently proposed Grey Wolf Optimizer (GWO). The contributions of this research are as follows:

- An archive has been integrated to the GWO algorithm to maintain non-dominated solutions.
- A grid mechanism has been integrated to GWO in order to improve the non-dominated solutions in the archive.
- A leader selection mechanism has been proposed based on alpha, beta, and delta wolves to update and replace the solutions in the archive.
- The multi-objective version of GWO has been proposed utilizing the above three operators.

The rest of the paper is organized as follows. Section 2 presents definitions and preliminaries of optimization in a multi-objective search space. Section 3 briefly reviews the concepts of GWO and then proposes the MOGWO algorithm. The qualitative and qualitative results as well as relevant discussion are presented in Section 4. Eventually, Section 5 concludes the work and outlines some advises for future works.

2. Literature review

This section provides the concepts of multi-objective optimization and current techniques in the field of meta-heuristics.

2.1. Multi-objective optimization

As briefly mentioned in the introduction, multi-objective optimization refers to the optimisation of a problem with more than one objective function. Without loss of generality, it can be formulated as a maximization problem as follows:

$$Maximize: \ F(\vec{x}) = f_1(\vec{x}), \ f_2(\vec{x}), \ \ldots, \ f_o(\vec{x}) \tag{2.1}$$

$$Subject\ to: \ g_i(\vec{x}) \geq 0, \ i = 1, 2, \ldots, m \tag{2.2}$$

$$h_i(\vec{x}) = \ 0, \ i = 1, 2, \ldots, p \tag{2.3}$$

$$L_i \ \leq x_i \leq U_i, \ i = 1, 2, \ldots, n \tag{2.4}$$

where n is the number of variables, o is the number of objective functions, m is the number of inequality constraints, p is the number of equality constraints, g_i is the ith inequality constraints, h_i indicates the ith equality constraints, and $[L_i,U_i]$ are the boundaries of ith variable.

In single-objective optimization, solutions can be compared easily due to the unary objective function. For maximization problems, solution X is better than Y if and only if $X > Y$. However, the solutions in a multi-objective space cannot be compared by the relational operators due to multi-criterion comparison metrics. In this case, a solution is better than (dominates) another solution if and only if it shows better or equal objective value on all of the objectives and provides a better value in at least one of the objective functions. The concepts of comparison of two solutions in multi-objective problems were first proposed by Francis Ysidro (Edgeworth, 1881) and then extended by Vilfredo Pareto (Pareto, 1964). Without loss of generality, the mathematical definition of Pareto dominance for a maximization problem is as follows (Coello, 2009):

Definition 1. Pareto Dominance:

Suppose that there are two vectors such as: $\vec{x} = (x_1, x_2, \ldots, x_k)$ and $\vec{y} = (y_1, y_2, \ldots, y_k)$.

Vector x dominates vector y (denote as $x \succ y$) *iff*:

$$\forall i \in \{1, 2, \ldots, k\}, \ [f(x_i) \geq f(y_i)] \wedge [\exists i \in 1, 2, \ldots, k : f(x_i)] \tag{2.5}$$

The definition of Pareto optimality is as follows (Ngatchou, Zarei, & El-Sharkawi, 2005):

Definition 2. Pareto Optimality:

A solution $\vec{x} \in X$ is called Pareto-optimal *iff*:

$$\nexists \vec{y} \in X \mid F(\vec{y}) \succ F(\vec{x}) \tag{2.6}$$

A set including all the non-dominated solutions of a problem is called Pareto optimal set and it is defined as follows:

Definition 3. Pareto optimal set:

The set all Pareto-optimal solutions is called Pareto set as follows:

$$P_s := \{x, y \in X \mid \exists F(y) \succ F(x)\} \tag{2.7}$$

A set containing the corresponding objective values of Pareto optimal solutions in Pareto optimal set is called Pareto optimal front. The definition of the Pareto optimal front is as follows:

Definition 4. Pareto optimal front: a set containing the value of objective functions for Pareto solutions set:

$$P_f := \{F(x) | x \in P_s\} \tag{2.8}$$