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Fuzzy interpolative reasoning for sparse fuzzy rule-based systems based on the slopes of fuzzy sets

Shyi-Ming Chen [a,*], Wen-Chyuan Hsin [a], Szu-Wei Yang [b], Yu-Chuan Chang [a]

[a] Department of Computer Science and Information Engineering, National Taiwan University of Science and Technology, Taipei, Taiwan, ROC
[b] Department of Education, National Taichung University of Education, Taichung, Taiwan, ROC

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ABSTRACT

In this paper, we present a new fuzzy interpolative reasoning method for sparse fuzzy rule-based systems based on the slopes of fuzzy sets. The proposed method can deal with fuzzy rules interpolation involving complex polygonal fuzzy sets with the advantages of simplest calculation and get more reasonable fuzzy interpolative reasoning results. We also make some experiments to compare the fuzzy interpolative reasoning results of the proposed method with the ones of the existing methods. The experimental results show that the proposed method outperforms the existing fuzzy interpolative reasoning methods for sparse fuzzy rule-based systems.

1. Introduction

In recent years, some fuzzy interpolative reasoning methods have been presented to overcome the drawback of spare fuzzy rule-base systems (Baranyi, Koczy, & Gedeon, 2004; Baranyi, Tikk, Yam, & Kozcy, 1999; Bouchon-Meunier, Marsala, & Rifqi, 2000; Chen & Chang, 2011; Chen, Hsin, & Chang, 2011; Chang, Chen, & Liau, 2008; Chen et al., 2011; Chen & Ko, 2008; Chen, Ko, Chang, & Pan, 2009; Chen & Lee, 2011; Hsiao, Chen, & Lee, 1998; Huang & Shen, 2003, 2006, 2008; Ko & Chen, 2007; Ko, Chen, & Pan, 2008; Koczy & Hirota, 1997, 1993a, 1993b; Koczy & Kovacs, 1994; Kong & Kosko, 1992; Lee & Chen, 2008; Li, Huang, Tsang, & Zhang, 2005; Tikk & Baranyi, 2000; Wong, Tikk, Gedeon, & Koczy, 2005; Yam, Baranyi, Tikk, & Koczy, 1999; Yam & Koczy, 2000; Yam, Wong, & Baranyi, 2006). Koczy and Hirota (1997) presented a method for approximate reasoning by linear-rule interpolation and general approximation (call the KH method) where based on the α-cuts of fuzzy sets, using the fuzzy distance between the observation and the antecedents of the fuzzy rules to get the fuzzy interpolative reasoning result. Chang et al. (2008) presented a fuzzy interpolate reasoning method for sparse fuzzy rule-base systems based on the areas of fuzzy sets (called the CCL method). Chen and Ko (2008) presented a fuzzy interpolative reasoning method (called the CK method) based on the increment transformation and the ratio transformation techniques for sparse fuzzy rule-based systems. Huang and Shen (2006) presented a fuzzy interpolative reasoning method via scale and the move transformation techniques (called the HS method).

In this paper, we present a new fuzzy interpolative reasoning method for sparse fuzzy rule-based systems based on the slopes of fuzzy sets. The proposed method can deal with fuzzy rules interpolation involving complex polygonal fuzzy sets with the advantages of simplest calculation and get more reasonable fuzzy interpolative reasoning results. We also make some experiments to compare the fuzzy interpolative reasoning results of the proposed method with the ones of the existing methods. The experimental results show that the proposed method outperforms the existing fuzzy interpolative reasoning methods for sparse fuzzy rule-based systems.

2. A new method for fuzzy interpolative reasoning based on the slopes of fuzzy sets

In this section, we present a new method for fuzzy interpolative reasoning based on the slopes of fuzzy sets (Zadeh, 1965).

2.1. Multiple antecedent variables fuzzy interpolative reasoning with bell-shaped membership functions

Assume that there are two fuzzy rules in the knowledge base of a rule-based system, shown as follows:

* Corresponding author. Tel.: +886 2 27376417; fax: +886 2 27301081.
E-mail address: smchen@mail.ntust.edu.tw (S.-M. Chen).

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Rule 1: IF $X_1 = A_{11}$ and $X_2 = A_{12}$ and ... and $X_m = A_{1m}$ THEN $Y = B_1$,
Rule 2: IF $X_1 = A_{21}$ and $X_2 = A_{22}$ and ... and $X_m = A_{2m}$ THEN $Y = B_2$,
Observation: X_1 is A_1^* and X_2 is A_2^* and ... and X_m is A_m^*

Conclusion: Y is B^*

where $X_1, X_2, \ldots$, and X_m are the antecedent variables of the fuzzy rules, A_{ij} denotes the jth antecedent fuzzy set of the fuzzy rule *Rule i*, B_i denotes the consequence fuzzy set of the fuzzy rule *Rule i*, $1 \leqslant i \leqslant 2$ and $1 \leqslant j \leqslant m$, where m denotes the number of antecedent variables. When the observation is "X_1 is A_1^* and X_2 is A_2^* and ... and X_m is A_m^*", then we can use the proposed method to derive the conclusion "Y is B^*". The membership function of a Bell-Shaped fuzzy set A is shown in Fig. 1, where $A = e^{-\left(\frac{x-c_A}{a_A}\right)^{2b_A}}$, a_A and b_A are the parameters controlling the shape of the bell-shaped fuzzy set A, and c_A is the mean of the bell-shaped fuzzy set A, respectively, and the Bell-Shaped fuzzy set A can be represented by $A = (a_A, b_A, c_A)$. Assume that $A_{ij} = (a_{A_{ij}}, b_{A_{ij}}, c_{A_{ij}})$, $B_i = (a_{B_i}, b_{B_i}, c_{B_i})$, $A_j^* = \left(a_{A_j^*}, b_{A_j^*}, c_{A_j^*}\right)$, $1 \leqslant j \leqslant m$, $1 \leqslant i \leqslant 2$ and $B^* = (a_{B^*}, b_{B^*}, c_{B^*})$, as shown in Fig. 2.

The proposed fuzzy interpolative reasoning method for sparse fuzzy rule-based systems using Bell-Shaped fuzzy sets is now presented as follows:

Step 1: Calculate the value of λ_{rep}, shown as follows:

$$\lambda_{rep} = \frac{d\left(A_{1j}, A_j^*\right)}{d(A_{1j}, A_{2j})} = \frac{\left|c_{A_j^*} - c_{A_{1j}}\right|}{\left|c_{A_{2j}} - c_{A_{1j}}\right|}. \tag{1}$$

Step 2: Calculate the parameters $a_{A'_j}$ and $b_{A'_j}$ of the fuzzy set $A'_j = (a_{A'_j}, b_{A'_j}, c_{A'_j})$ and a'_B and b'_B of the fuzzy set $B' = (a'_B, b'_B, c'_B)$, respectively, shown as follows:

$$a_{A'_j} = (1 - \lambda_{rep}) \times a_{A_{1j}} + \lambda_{rep} \times a_{A_{2j}}, \tag{2}$$

$$b_{A'_j} = (1 - \lambda_{rep}) \times b_{A_{1j}} + \lambda_{rep} \times b_{A_{2j}}, \tag{3}$$

$$a'_B = (1 - \lambda_{rep}) \times a_{B_1} + \lambda_{rep} \times a_{B_2}, \tag{4}$$

$$b'_B = (1 - \lambda_{rep}) \times b_{B_1} + \lambda_{rep} \times b_{B_2}. \tag{5}$$

where $1 \leqslant j \leqslant m$.

Step 3: Calculate the values of a_{B^*} and b_{B^*} of the fuzzy set B^*, respectively, shown as follows:

$$\frac{a_{B^*}}{a'_B} = \frac{1}{m}\left(\sum_{j=1}^{m} \frac{a_{A_j^*}}{a_{A'_j}}\right), \tag{6}$$

$$a_{B^*} = a'_B \times \frac{1}{m}\left(\sum_{j=1}^{m} \frac{a_{A_j^*}}{a_{A'_j}}\right), \tag{7}$$

$$\frac{b_{B^*}}{b'_B} = \frac{1}{m}\left(\sum_{j=1}^{m} \frac{b_{A_j^*}}{b_{A'_j}}\right), \tag{8}$$

$$b_{B^*} = b'_B \times \frac{1}{m}\left(\sum_{j=1}^{m} \frac{b_{A_j^*}}{b_{A'_j}}\right). \tag{9}$$

Step 4: Calculate the means $c_{A'_j}$ and c'_B of the fuzzy sets $A'_j = (a_{A'_j}, b_{A'_j}, c_{A'_j})$ and $B' = (a'_B, b'_B, c'_B)$, respectively, where

$$c_{A'_j} = (1 - \lambda_{rep}) \times c_{A_{1j}} + \lambda_{rep} \times c_{A_{2j}}, \tag{10}$$

$$c'_B = (1 - \lambda_{rep}) \times c_{B_1} + \lambda_{rep} \times c_{B_2} \tag{11}$$

where $1 \leqslant j \leqslant m$.

Step 5: Calculate the mean c_{B^*} of the fuzzy set $B^* = (a_{B^*}, b_{B^*}, c_{B^*})$, shown as follows:

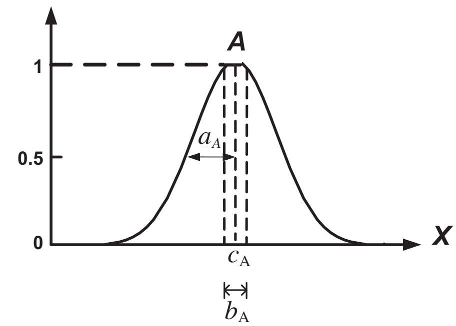

Fig. 1. A Gaussian fuzzy set.

$$\frac{1}{m}\sum_{j=1}^{m} \frac{c_{A_j^*} - c_{A'_j}}{c_{A_{2j}} - c_{A_{1j}}} = \frac{c_{B^*} - c'_B}{c_{B_2} - c_{B_1}}, \tag{12}$$

$$c_{B^*} = c'_B + (c_{B_2} - c_{B_1}) \times \frac{1}{m}\sum_{j=1}^{m} \frac{c_{A_j^*} - c_{A'_j}}{c_{A_{2j}} - c_{A_{1j}}}. \tag{13}$$

Based on the values of a_{B^*}, b_{B^*} and c_{B^*}, the Bell-Shaped membership function of the interpolated consequence fuzzy set $B^* = (a_{B^*}, b_{B^*}, c_{B^*})$ is derived.

2.2. Multiple multiantecedent rules fuzzy interpolation scheme with polygonal membership functions

Assume that there are n fuzzy rules in the knowledge base of a rule-based system, shown as follows:

Rule 1: IF $X_1 = A_{11}$ and $X_2 = A_{12}$ and ... and $X_m = A_{1m}$ THEN $Y = B_1$,
Rule 2: IF $X_1 = A_{21}$ and $X_2 = A_{22}$ and ... and $X_m = A_{2m}$ THEN $Y = B_2$,
⋮
Rule n: IF $X_1 = A_{n1}$ and $X_2 = A_{n2}$ and ... and $X_m = A_{nm}$ THEN $Y = B_n$,
Observation: X_1 is A_1^* and X_2 is A_2^* and ... and X_m is A_m^*

Conclusion: Y is B^*

where $X_1, X_2, \ldots$, and X_m are the antecedent variables of the fuzzy rules, A_{ij} denotes the jth antecedent fuzzy set of *Rule i*, B_i denotes the consequence fuzzy set of *Rule i*, $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant m$, n denotes the number of fuzzy rules and m denotes the number of antecedent variables. If the observation is "X_1 is A_1^* and X_2 is A_2^* and ... and X_m is A_m^*", then we can use the propose method to derive the conclusion "Y is B^*". Based on the operations of α-cuts, a polygonal fuzzy set A can be characterized by n points $a_1, a_2, \ldots$ and a_n, as shown in Fig. 3 (Chang et al., 2008), where $A = (a_1, a_2, \ldots a_n)$, $a_{\lfloor n/2 \rfloor}$ and $a_{\lceil n/2 \rceil}$ are called the "left normal point" and the "right normal point", respectively, and a_1 and a_n are call the "left extreme point" and the "right extreme point", respectively (Chang et al., 2008), and $h(a_i)$ denote the membership value of a_i in the polygonal fuzzy set A, where $0 \leqslant h(a_i) = h(a_{n+1-i}) \leqslant 1$. In (Huang & Shen, 2006), Huang and Shen presented three kinds of representative values of polygonal fuzzy sets, i.e., "the general rep", "the weighted average rep" and "the center of core rep". The general rep $rep_G(A)$ of the polygonal fuzzy sets $A = (a_1, a_2, \ldots, a_n)$ shown in Fig. 3 is calculated as follows (Huang & Shen, 2006):

$$rep_G(A) = \sum_{i=1}^{n} w_i a_i, \tag{14}$$