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Combining prospect theory and fuzzy numbers to multi-criteria decision making

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ABSTRACT

Many multi-criteria decision making (MCDM) methods have been proposed to handle uncertain decision making problems. Most of them are based on fuzzy numbers and they are not able to cope with risk in decision making. In recent years, some MCDM methods based on prospect theory to handle risk MCDM problems have been developed. In this paper, we propose a hybrid approach combining prospect theory and fuzzy numbers to handle risk and uncertainty in MCDM problems. So, it is possible to tackle more challenging MCDM problems. A case study involving oil spill in the sea illustrates the application of the novel method.

1. Introduction

Complex decision processes may be considered difficult to solve most due to the involved uncertainties, associated risks and inherent complexities of multi-criteria decision making (MCDM) problems (Fenton & Wang, 2006). Methods for solving MCDM problems e.g. TOPSIS (Hwang & Yoon, 1981), PROMETHEE (Brans, Vincke, & Mareschal, 1986) have been widely used to select the best alternative among a finite number of alternatives generally characterized by multiple criteria (attributes). Despite their usefulness, these methods are often criticized because of their inability to deal adequately with uncertainty and imprecision inherent in the process of mapping the perceptions of decision-makers. In the standard formulation of TOPSIS and PROMETHEE, the personal judgments are represented by numerical values. However, in many cases the human preference model is uncertain and the decision-makers might be unable to assign numerical values to the judgments of comparison.

The theory of fuzzy sets and fuzzy logic developed by Zadeh (1965) has been used to model uncertainty or lack of knowledge and applied to a variety of MCDM problems. Bellman and Zadeh (1970) introduced the theory of fuzzy sets in problems of MCDM as an effective approach to treat vagueness, lack of knowledge and ambiguity inherent in the human decision making process which are known as fuzzy multi-criteria decision making (FMCDM). See for example Zimmermann (1991) for more information.

For real world-problems the decision matrix is affected by uncertainty and may be modeled using fuzzy numbers. A fuzzy number (Dubois & Prade, 1980) can be seen as an extension of an interval with varied grade of membership. This means that each value in the interval has associated a real number that indicates its compatibility with the vague statement associated with a fuzzy number. So, standard MCDM methods like TOPSIS and PROMETHEE have been extended using fuzzy numbers resulting in fuzzy TOPSIS (Wang, Liu, & Zhang, 2005) and fuzzy PROMETHEE (Goumas & Lygerou, 2000), respectively. Both methods have successfully been applied to solve various uncertain MCDM problems.

Another important aspect of decision making is that most of the existing MCDM methods are not able to capture or take into account the risk attitude/preferences of the decision maker in MCDM. Prospect theory developed by Kahneman and Tversky (1979) is a descriptive model of individual decision making under condition of risk. Later, Tversky and Kahneman (1992) developed the cumulative prospect theory, which capture psychological aspects of decision making under risk. In the prospect theory, the outcomes are expressed by means of gains and losses from a reference alternative (Salminen, 1994). The value function in prospect theory assumes a S-shape concave above the reference alternative, which reflects the aversion of risk in face of gains; and the convex part below the reference alternative reflects the propensity to risk in case of losses.

Prospect theory has successfully been used as behavioral model of decision making under risk mainly in economics and finance (Dhami & Al-Nowaihi, 2007; Gurevich, Kliger, & Levy, 2009). Unfortunately, little research works have reported the application of prospect theory to MCDM problems. As far as we know, one of the first MCDM methods based on prospect theory was proposed by Gomes and Lima (1992). Despite its effectiveness and simplicity in concept, this method presents some shortcomings because of its inability to deal with uncertainty and imprecision inherent in the

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process of decision making. In the original formulation of TODIM (an acronym in Portuguese for Iterative multi-criteria decision making), the rating of alternatives, which composes the decision matrix, is represented by crisp values. The TODIM method has many similarities with the PROMETHEE method, whereas the preference function is replaced by the prospect function. The TODIM method has been applied to rental evaluation of residential properties (Gomes & Rangel, 2009) among others applications with good performance. However, one of the shortcomings of the TODIM method is its inability to handle uncertainty, which is present in many MCDM problems.

In the meantime, Wang and Sun (2008) proposed a MCDM method inspired by prospect theory combined with fuzzy PROMETHEE to handle uncertain decision making problems. The authors developed a method, whereas a prospect value function of trapezoidal fuzzy numbers based on risk attitudes of decision makers is defined, and the preference function in PROMETHEE method is replaced by the possibility degree of prospect value function of trapezoidal fuzzy numbers. Their preliminary results on a single uncertain and risk MCDM shows the feasibility of their method and seems to be promising. Recently, Liu, Jin, Zhang, Su, and Wang (2011) developed an approach where the prospect value of attribute for every alternative is calculated through a prospect value function of the trapezoidal fuzzy number and the weight function of interval probability. So, the weighted prospect value of the alternatives is obtained by using the weighted average method according to attribute weights, and all the alternatives are sorted according to the expected values of the weighted prospect values. The authors report the results for MCDM concerned with an investment company and show the feasibility of their method.

In this paper, we combine the strong aspects of prospect theory and fuzzy numbers to handle risk and uncertain MCDM problems. The goal consists in the extension of TODIM (Gomes & Rangel, 2009) to handle uncertain decision matrices resulting in Fuzzy TODIM. In Section 2, we shortly describe the TODIM method. In Section 3, we propose the novel Fuzzy TODIM method, which contains uncertainty in the decision matrix using trapezoidal fuzzy numbers and the value function of prospect theory. In Section 4, we present a case study to illustrate the method and results show the feasibility of the approach. In Section 5, we present some conclusions and directions for future work.

2. Multi-criteria decision making based on prospect theory

2.1. Preliminaries on prospect theory

The value function used in the prospect theory is described in form of a power law according to the following expression (Kahneman & Tversky, 1979):

$$v(x) = \begin{cases} x^{\alpha} & \text{if } x \geqslant 0 \\ -\theta(-x)^{\beta} & \text{if } x < 0 \end{cases} \tag{1}$$

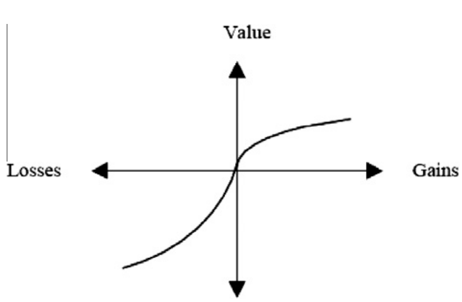

Fig. 1. Value function of prospect theory.

where α and β are parameters related to gains and losses, respectively. The parameter θ represents a characteristic of being steeper for losses than for gains. In case of risk aversion $\theta > 1$. Fig. 1 shows a prospect value function with a concave and convex S-shaped for gains and losses, respectively. Kahneman and Tversky (1979) experimentally determined the values of $\alpha = \beta = 0.88$, and $\theta = 2.25$, which are consistent with empirical data. Further, they suggest that the value of θ is between 2.0 and 2.5.

2.2. Multi-criteria decision making – the TODIM method

Let us consider the decision matrix A, which consists of *alternatives* and *criteria*, described by:

$$A = \begin{array}{c} \\ A_1 \\ \cdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \cdots & C_n \end{array} \\ \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \end{array}$$

where $A_1, A_2, \ldots, A_m$ are viable alternatives, and $C_1, C_2, \ldots, C_n$ are criteria, x_{ij} indicates the rating of the alternative A_i according to criteria C_j. The weight vector $W = (w_1, w_2 \ldots, w_n)$ composed of the individual weights w_j ($j = 1, \ldots, n$) for each criterion C_j satisfying $\sum_{j=1}^{n} w_j = 1$. The data of the decision matrix A come from different sources, so it is necessary to normalize it in order to transform it into a dimensionless matrix, which allow the comparison of the various criteria. In this work, we use the normalized decision matrix $R = [r_{ij}]_{m\times n}$ with $i = 1, \ldots, m$, and $j = 1, \ldots, n$. After normalizing the decision matrix and the weight vector, TODIM begins with the calculation of the partial dominance matrices and the final dominance matrix. For such calculations the decision makers need to define firstly a reference criterion, which usually is the criterion with the highest importance weight. So, w_{rc} indicates the weight of the criterion c divided by the reference criterion r. Basically, TODIM is described in the following steps (Gomes & Lima, 1992; Gomes & Rangel, 2009):

Step 1: Calculate the dominance of each alternative A_i over each alternative A_j using the following expression:

$$\delta(A_i, A_j) = \sum_{c=1}^{m} \phi_c(A_i, A_j) \quad \forall (i,j) \tag{2}$$

where

$$\phi_c(A_i, A_j) = \begin{cases} \sqrt{\dfrac{w_{rc}(x_{ic}-x_{jc})}{\sum_{c=1}^{m} w_{rc}}} & \text{if } (x_{ic} - x_{jc}) > 0 \\ 0 & \text{if } (x_{ic} - x_{jc}) = 0 \\ \dfrac{-1}{\theta}\sqrt{\dfrac{\left(\sum_{c=1}^{m} w_{rc}\right)(x_{jc}-x_{ic})}{w_{rc}}} & \text{if } (x_{ic} - x_{jc}) < 0 \end{cases} \tag{3}$$

The term $\phi_c(A_i, A_j)$ represents the contribution of the criterion c to the function $\delta(A_i, A_j)$ when comparing the alternative i with alternative j. The parameter θ represents the attenuation factor of the losses, which can be tuned according to the problem at hand. In Eq. (3) it can occur three cases: (i) if the value $(x_{ic} - x_{jc})$ is positive, it represents a gain; (ii) if the value $(x_{ic} - x_{jc})$ is nil; and (iii) if the value $(x_{ic} - x_{jc})$ is negative, it represent a loss. The final matrix of dominance is obtained by summing up the partial matrices of dominance for each criterion.

Step 3: Calculate the global value of the alternative i by normalizing the final matrix of dominance according to the following expression:

$$\xi_i = \frac{\sum \delta(i,j) - \min \sum \delta(i,j)}{\max \sum \delta(i,j) - \min \sum \delta(i,j)} \tag{4}$$