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A weighted inference engine based on interval-valued fuzzy relational theory

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ABSTRACT

The study of fuzzy relations forms an important fundamental of fuzzy reasoning. Among all, the research on compositional fuzzy relations by Bandler and Kohout, or the Bandler–Kohout (BK) subproduct gained remarkable success in developing inference engines for numerous applications. Despite of its successfulness, we notice that there are limitations associated in the current implementations of the BK subproduct. In this paper, the BK subproduct, which originally based on the ordinary fuzzy set theory, is extended to the interval-valued fuzzy sets. This is because studies had claimed that ordinary fuzzy set theory has its limitation in addressing uncertainties using the crisp membership functions. Secondly, with the understanding that some features might have higher influence compare to the others, a weight parameter is introduced in the BK subproduct-based inference engines. Finally, a fuzzification method that able to fuzzify the input data and also train the inference engines is also developed. So, the BK subproduct-based inference systems can be built without human intervention, which are cumbersome and time consuming. Experiments on three public datasets and a comparison with state-of-art solutions have shown the efficiency and robustness of the proposed method.

1. Introduction

Reasoning with fuzzy sets theory has been widely studied in the literature. Some common directions of studies include finding soft cluster centers of data (Roh, Pedrycz, & Ahn, 2014; Son, 2015), integrate the fuzzy sets theory with other technologies such as artificial neural networks (Egrioglu, Aladag, & Yolcu, 2013; Suparta & Alhasa, 2014) or particle swarm optimization (Melin et al., 2013), modeling parameters as fuzzy numbers (Petrović et al., 2014; Samantra, Datta, & Mahapatra, 2014) and etc (Abdullah & Najib, 2014; Nguyen, Dawal, Nukman, & Aoyama, 2014). Among all, many researches focus on constructing fuzzy inference systems for various purposes, such as classification (Ait Laasri, Akhouayri, Agliz, Zonta, & Atmani, 2015; D'Andrea & Lazzerini, 2013; Samuel, Omisore, & Ojokoh, 2013; Yuste, Triviño, & Casilari, 2013), control (Bugarski, Bačkalić, & Kuzmanov, 2013; Liu, Han, & Lu, 2013; Liu, Yang, & Yang, 2013) and etc (Camastra et al., 2015; Gupta, Saini, & Saxena, 2014). In all these fuzzy inference systems, fuzzy rules are the core of the inference processes.

However, the popularity of these rule based fuzzy inference systems dispute the necessity of developing other inference schemes. Among all, a theory on compositions of fuzzy relations, namely the Bandler-Kohout (BK) subproduct (Kohout & Bandler, 1980a, 1980b) shows its excellency in some studies (Bodenhofer, Dankova, Stepnicka, & Novak, 2007; Stepnicka & Jayaram, 2010). Compare to those popular rule based fuzzy inference systems, a special characteristic of BK subproduct based inference systems is it does not require rules. Certainly, it is an advantage in the cases which rules are hardly define (Kolodner, 1992). While some theoretical researches on BK subproduct are in advancement (Mandal & Jayaram, 2013; Stepnicka & De Baets, 2013; Štěpnička & Holčapek, 2014), empirical works have also been carried out to prove its advantages in many expert systems, such as medical diagnostic systems (Kohout, Stabile, Kalantar, & San-Andres, 1995; Yew & Kohout, 1997), information retrieval (Kohout & Bandler, 1985), land evaluation (Groenemans, Ranst, & Kerre, 1997), cognitive sciences (Kohout, 2009), scene understanding (Vats, Lim, & Chan, 2012) and etc.

Despite of its successfulness, we notice that there are still some limitations associated in the current implementations of the BK subproduct. First of all, the implementations of the BK subproduct in the literature are still based on the classical Type-1 Fuzzy Set (T1FS) theory, which address uncertainties with point-values. Studies such as Mendel (2000, 2003) claimed that T1FS has its limitation in addressing uncertainties with its crisp membership functions. Secondly, the BK subproduct performs inferences utilize a set

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of common features that relate the inputs and outputs. In most cases, the BK subproduct treats all the features equally, i.e. the importance of all the features are similar. However, in practical, not all these features have the identical influences towards the inference outcomes. We argue that some features may have higher reliability or distinguishability than the others, and vice versa. In the literature of fuzzy logic, implementation of the weight parameter is not rare (Ishibuchi & Yamamoto, 2005; Seki & Mizumoto, 2011; Xing & Ha, 2014). For example, Groenemans et al. (1997) have tried to incorporate the weight parameter in the BK subproduct. However, this implementation would require to fulfill a condition: $\sum_{n=1}^{N} w_n = 1$ where $n = \{1, 2, \ldots, N\}, N$ is the number of features and w_n is the weight of feature n. This condition is too restrictive for a good implementation of weight because: *(i)* adding or decreasing features into consideration list will cause recalculation of all the weights. For instance, adding a new feature with weight $w_{N+1} \neq 0$ to the existing feature list will cause the total weight become $\sum_{n=1}^{N+1} w_n > 1$ and the condition of total weight equal to 1 is not longer hold. Thus, a normalization is required so that the $\sum_{n=1}^{N+1} w_n = 1$ is satisfied. *(ii)* importance or influence of a feature is not intuitive – i.e. comparing a system with such condition to an implementation of weight where $w_n \in [0, 1]$ for all n, the weights of the latter are much intuitive as the weights close to 0 means less influence, while close to 1 means high influence. Groenemans et al. (1997) found out that the weights can be small numbers that close to 0 even if they have high influence in the case of number of features N is large. Furthermore, this problem become much more complicated if new features that are going to be added into consideration, as one may not know what are the appropriate values that representing the high (or low) influence.

Last but not least, in all the previous attempts of the BK subproduct, predefine rules (Bui & Kim, 2006) or experts knowledge (Kohout et al., 1995) are priori required, so that the knowledge base can be formed. In some cases, worst still the fuzzification of the input data are done manually, which is cumbersome and time consuming. An approach to train the BK subproduct automatically so that it can learn from the training examples could not be found in the literature, and the lack of this learning mechanism greatly limits the application of the BK subproduct in many fields.

Hence, the aims of this paper is to form a more reliable BK subproduct reasoning framework. For this purpose, our contributions are threefolds: (1) we extend the current BK subproduct to the Interval-Valued Fuzzy Sets (IVFS) by defining a subsethood measure of IVFS. (2) a weight parameter is incorporated to the BK subproduct-based inference engines so that more attention is given to those important features, and (3) we introduce a learning mechanism for the BK subproduct. Employing the training samples, the proposed learning mechanism manages to form the knowledge base of a BK subproduct-based inference engine without human intervention. Additionally, the learning mechanism also produce membership functions that serve to fuzzify the input data. To prove the advantages and improvements of the proposed BK subproduct, three publicly available medical data sets are employed. Experimental results and a comparison with state-of-the-art solutions have shown the efficiency of the proposed method.

The rest of the paper are arranged as follow: In Section 2, we provide a short revision on the BK subproduct. Section 3 discusses the extension of BK subproduct from T1FS to IVFS, along with a subsethood measure of IVFS. In Section 4, we introduce the weight parameter to this newly developed IVFS reasoning scheme. Section 5 proposes a learning mechanism so that the BK subproduct based inference systems can be built from numerical data. The classification experiment and the discussion is presented in Section 6. Lastly, we conclude the paper in Section 7.

2. BK subproduct revisit

We start this section with a brief review on the definition of the BK subproduct on crisp relations. Assume that A, B and C are three crisp sets and a, b and c are general representation to the elements in these sets respectively. R is defined as a relation from A to B such that $R \subseteq A \times B$; whereas S is a relation from B to C such that $S \subseteq B \times C$. The converse relation of S is denoted as S^T. The abbreviation aRb shows that a is in relation R with b. Kohout and Bandler (1980a) defined the BK subproduct as follow:

Definition 1. The BK subproduct is a composition of relations for a and c such that:

$$R \triangleleft S = \{(a, c) | (a, c) \in A \times C \quad \text{and} \quad aR \subseteq Sc\} \tag{1}$$

where $aR = \{b | aRb\}$ is the image of a after the projection of relation R in the set B, while $Sc = \{b | bSc\}$ is the image of c after the projection of relation S^T in the set B. The BK subproduct is useful in retrieving relationships between elements of two indirectly associated sets, *objects* A and *targets* C, if both sets can be associated with a set of common *features*, B.

It is trivial that Definition 1 is established on the subsethood of aR in Sc. In order to extend it to the fuzzy relations, the fuzzy subsethood measure for T1FS is defined as:

Definition 2. For two T1FS, P and Q in the same universe X, the possibility of P is a subset of Q is given as follow:

$$\pi(P \subseteq Q) = \bigwedge_{x \in X} (P(x) \rightarrow Q(x)) \tag{2}$$

where $P(x)$ and $Q(x)$ are the membership degrees of x in set P and Q respectively, $\rightarrow$ is the fuzzy implication operators (see Table 1) generally defined as "NOT A OR B" and $\wedge$ is the infimum operator which can be considered as min function in harsh criterion or arithmetic mean in mean criterion (Kohout & Bandler, 1980a).

Definition 3. Employing the Eqs. (1) and (2), Kohout and Bandler (1980a) defined the fuzzy BK subproduct as follow:

$$R \triangleleft S(a, c) = \bigwedge_{b \in B} (R(a, b) \rightarrow S(b, c)) \tag{3}$$

where $R(a, b)$ is the fuzzy membership degree to which aRb is true and $S(b, c)$ is the fuzzy membership degree to which bSc is true.

Table 1
Example of the fuzzy implication operators and their respective definitions.

Name	Symbol	Definition
S# – Standard Sharp (Mizumoto & Zimmermann, 1982)	$r \rightarrow_{S\#} s$	$\begin{cases} 1 & \text{iff } r \neq 1 \text{ or } s = 1 \\ 0 & \text{otherwise} \end{cases}$
S – Standard Strict (Mizumoto & Zimmermann, 1982)	$r \rightarrow_{S} s$	$\begin{cases} 1 & \text{iff } r \leqslant 1 \\ 0 & \text{otherwise} \end{cases}$
G43 – Gaines 43 (Mizumoto & Zimmermann, 1982)	$r \rightarrow_{G43} s$	$\min(1, \frac{r}{s})$
KD – Kleene-Dienes (Kohout & Bandler, 1980a)	$r \rightarrow_{KD} s$	$\max(s, 1 - r)$
R – Reichenbach (Kohout & Bandler, 1980a)	$r \rightarrow_{R} s$	$1 - r + rs$
L – Łukasiewicz (Zadeh, 1975)	$r \rightarrow_{Ł} s$	$\min(1, 1 - r + s)$
Y – Yager (Yager, 1980)	$r \rightarrow_{Y} s$	s^r
EZ – Early Zadeh (Zadeh, 1975)	$r \rightarrow_{EZ} s$	$(r \wedge s) \vee (1 - r)$