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# Risk-based multivariate control chart

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ABSTRACT

Control charts are widely used tools in statistical process control (SPC). Most of the control charts operates on reliability base, so the users assume that the value of the real product characteristic is equal to the value derived from the measurement. It is a frequent case that the conformity of a product is determined by more than one product characteristics. It is recommended to apply multivariate control charts for the control of multiple product characteristics, but the measurement uncertainty can lead to incorrect decisions even in univariate case. In this study, the authors develop a risk-based multi-dimensional $T^2$ chart ($RBT^2$), which takes the consequences of the decisions into account and reduces the risks during the process control. The proposed method can be applied even for non-normally distributed data. Several sensitivity analyses are provided and the performance of the $RBT^2$ chart is demonstrated when Six Sigma regulations are fulfilled.



## 1. Introduction

### 1.1. SPC and multivariate charts

The statistical process control is a widely used method, where the major tools are the control charts. Applying this method, the assignable causes could be found and corrected (Montgomery, 2012; Woodall & Montgomery, 1999). The most companies apply a lot of processes and subprocesses which need to be statistically controlled to guarantee the high quality of the product. When the expected value of the process changes significantly, the chart detects it and gives a signal for the operators. In statistical process control, a process is said to be controlled, when the observed values of product characteristic fall within the given limits (Besterfield, 1994; Shewhart, 1931). In most industries, univariate control charts are applied for monitoring process expected value and variance. At first A. W. Shewhart developed an X-bar control chart in 1924, to monitor the product characteristic.

At the beginning, univariate control charts were solely used for monitoring the process variation. Nevertheless, it is a frequent case that the conformity of a product is determined by more than one product parameters. If the process variables are dependent on each other, the application of more univariate control charts can rise the false alarm rate significantly. Though A.W. Shewhart dealt with monitoring of more correlated characteristics, the multivariate control chart has its origins in the work of H. Hotelling, who developed the $T^2$ chart based on Student's t-distribution (Hotelling, 1947). Other typical multivariate control charts are the multivariate sum (MCUSUM) control chart (Croiser, 1988; Pignatiello & Runger, 1990), and the exponentially weighted moving average chart (MEWMA), developed by (Lowry, Woodall, Champ, & Rigdon, 1992). Multivariate control chars are not only useful for monitoring product characteristics but even can be applied by delivery chains. Several references give a more detailed discussion about the multivariate quality control reviewed by Jackson (1985). Several papers discuss the design of principal component analysis based multivariate control chart, and $T^2$ chart with variable sampling interval (Chen, 2007; Chou, Chen, & Chen, 2006; Phaladiganon, Kim, Chen, & Jiang, 2013). However these multivariate control charts can detect a process shift, but they do not determine which one of the monitored variables (or which subset of characteristics) causes the out-of control signal. A number of references have proposed decomposition techniques (See: Hawkins, 1993; Hayter and Tsui, 1994; Lieftucht, Kruger, and W., 2006; Mason, Tracy, and Young, 1997 ) for the determination of the responsible characteristic(s).

In this study we assume, that the process shift is correctly detectable. This paper focuses on the effect of measurement uncertainty in multivariate process control.

Though, the application of the Hotelling's $T^2$ chart is an effective tool to monitor multivariate processes, the chart cannot be applied if the data are non-normally distributed. Several studies de-

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veloped different approaches to handle this issue (See: Bakir, 2006; Chakraborti, Laan, & Bakir, 2001; Chakraborti, van der Laan & van de Wiel, 2006; Tuerhong, Kim, Kang, & Cho, 2014).

The non-normality is not the only factor, that can reduce the effectiveness of the multivariate control charts. Most control charts assume that the detected values do not contain any measurement error, however the measurement uncertainty can cause serious problems during the application of the statistical process control. The aim of this study is to develop a novel multivariate control chart, that reduces the number of incorrect decisions (taking the measurement uncertainty into account).

### *1.2. Measurement uncertainty in statistical process control*

The definition of measurement uncertainty is bound up with the statistical process control. The ISO/BIPM "Guide to Expression of Uncertainty in Measurement" briefly referred as GUM (or Guide) was published in 1993, corrected and reprinted in 1995. It contains detailed examples from measurement problems and defines the term "measurement uncertainty".

There are many discussions about the customer's and supplier's risks (Lira, 1999) associated with the incorrect decisions arising from the inaccuracy of measurement (Pendrill, 2008). This uncertainty can lead to unnecessary or missed interventions during the application of the control charts, which increases the rate of supplier's or customer's risk.

The correctness of the conformity decisions (Is the product conformable or not?) also depends on the accuracy of the measured values. Suppose a process characteristic denoted with $x$ and $USL$, $LSL$ are the upper- and lower specification limits. The product is accepted if $LSL \leq x \leq USL$, otherwise it is rejected. The product characteristic is measured, and the measurement result is denoted with $y$. The detected value can be described with the following form:

$$y_i = x_i + \varepsilon_i \tag{1}$$

Where $\varepsilon_i$is the measurement error. The decision is incorrect in the following two cases:

incorrect rejecting:

$$x_i \in [LSL, USL] \text{ and } y_i \notin [LSL, USL] \tag{2}$$

incorrect accepting:

$$x_i \notin [LSL, USL] \text{ and } y_i \in [LSL, USL] \tag{3}$$

In the other two cases, the decisions are correct The commission of the Type II error can lead to serious consequences, because the selling of a non-conformable product can cause prestige loss for a manufacturer company (especially in the automotive- or medicine industry).

By the expression of the measurement uncertainty the expected uncertainty (denoted by $U$) can be obtained by multiplying the standard uncertainty ($u_c(y)$) by a $k$ coverage factor. ($U = ku_c(y)$).Several industrial standards recommend a method for the control chart applications, which means the modification of the $k$ coverage factor. For reviews of recommended coverage factor modifications the reader is referred to (EA, 2003; Eurachem, 2007; Eurolab, 2006; Hässelbarth, 2006; ILAC, 2009; NIST, 1994). A given value of $k$ can be applied if the distribution of the measurement error is symmetric. In many cases, the probability distribution of the measurement error is derived from more different types.

The multivariate control charts are effective tools for monitoring multivariate processes,however they disregard the measurement uncertainty. This paper focuses on the development of the Hotelling's T$^2$ control chart, which is an effective tool for multiple parameter-monitoring, but it has some weaknesses.

- The applicability of the chart is restricted in situations where the data are non-normally distributed.
- The Hotelling's T$^2$ chart does not take the consequences of the decisions into account during the conformity testing.
- The chart does not consider the measurement uncertainty during the monitoring process.

The aim of this study is the development of a new Risk-Based T$^2$ chart that is able to handle the three problems above. The significance of this research is a new Risk-Based Multivariate control chart which considers also the consequences of the decisions during the conformity control and can be applied even for non-normally distributed data. This paper also highlights the importance of the effect of the measurement uncertainty and suggests a method for the decision maker to reduce the decision risks during the process control. Furthermore, this study demonstrates the performance and sensitivity of the proposed RBT$^2$ chart when the Six Sigma regulations are fulfilled.

During the construction of the RBT$^2$ chart the control limit is modified with a correction component $K$. The value of the correction component has to be optimized. For the optimization, the Nelder-Mead algorithm is applied, which was developed by Nelder and Mead in 1965. This algorithm is one of the most popular direct search methods (Barton & Ivey, 1996). Furthermore, with sufficient geometrical justifications, the Nelder-Mead algorithm can find the improved solution more efficiently than the random search (Chang, 2012). This method easily expanded with more parameters during the minimization problem and also can be applied with different kind of control charts.

Dunchan was the first, who proposed an economic design model which determines the test parameters of an X-bar chart that minimizes the average cost, when an out of control state occurs. There are many studies with respect to the economic design of univariate and multivariate control charts (Celano, 2011; Franco, Costa, & Machado, 2012; Ho & Case, 2014; Montgomery, 1980; Torng, Lee, S., & Y., 2009) In these studies, the control charts are used with the goal to minimize the cost function. The economic design focuses on the costs related to the process control ( e.g.: cost of sampling, cost of the search of the problem), the proposed method focuses on the costs of decision consequences. The economic design approach contains the determination of the control chart parameters (e.g.: sample size, control limits, sampling interval) with a cost minimization method. In contrast, the Risk-Based control chart approach focuses only on the consequences of the decisions (described by cost) during the control process. However, this paper does not deal with the economic design of the Risk-Based T$^2$ chart, the method can be applied with economic design model in a further research.

Section 2 describes the Hotelling's T$^2$ chart in detail. Section 3 introduces the decision cost model. Section 4 presents the construction method of the Risk-Based T$^2$ chart (RBT$^2$). Section 5 compares the performance of the proposed control chart with the Hotelling's T$^2$ chart. A sensitivity analysis is provided in Section 6. Section 7 introduces the performance and the sensitivity of the proposed method by the measurement system capability. Section 8 summarizes the conclusions of this paper.

## 2. The reliability based T$^2$ chart

Hotelling's T$^2$ chart is the multivariate extension of the univariate Shewhart control chart. Assume that $X_i, i = 1, 2, 3, \ldots$ vector is representing the $p$ quality characteristics of a given product, furthermore assume that the $p$ characteristics have a $p$-variate normal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. If the process parameters are known, the T$^2$ statistic follows a chi-square distribution:

$$\chi_i^2 = n((\overline{\mathbf{X}}_i) - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}((\overline{\mathbf{X}}_i) - \boldsymbol{\mu}) \tag{4}$$