Biomechanics of the lower limb

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Abstract

This article outlines concepts of mechanics used in orthopaedics. These concepts are then demonstrated (assuming only a basic understanding of physics) with relevance to the hip, knee, ankle and foot and used to explain some common conditions. Some equations are used in this article for completeness; they are not essential to understanding the core principles.

Keywords Biomechanics; hip; knee; mechanics; statics

Introduction

Mechanics is the branch of physics that describes how a structure behaves when subjected to loading and/or motion. **Statics** is the branch of mechanics that considers structures at rest, or travelling at constant velocity in a straight line (all of the forces acting on the structure are considered to be in equilibrium; the net force acting on the structure is zero). **Dynamics** is the branch of mechanics which deals with the motion of bodies under the action of forces. Dynamics has two distinct parts; kinematics, which is the study of the motion itself (without reference to the forces) and kinetics, which focuses on the forces causing the motion. Dynamics is a relatively recent subject compared with statics. Galileo (1564–1642) is credited with its inception; by contrast, statics were well understood by early Greek scholars like Archimedes (287BC–212BC). **Biomechanics** is the study of mechanical laws relating to the movement or structure of living organisms.

This article will focus on statics.

Rigid body statics

Considering objects to be rigid bodies simplifies static analyses as it implies they cannot deform. A 'free body diagram' of the object is a pictorial representation showing all the forces acting on it. We will use as an example the forces acting at the hip joint but first clarify some terminologies:

1. **Force:** a vector quantity with magnitude and direction (the direction can be resolved into mutually perpendicular directions);
2. **Moment:** the tendency of a force to rotate an object about a pivot point. Thus a moment is the applied force, $\mathbf{F}$, multiplied by the perpendicular distance, $\boldsymbol{d}$, between the force and the pivot point ($\mathbf{M} = \mathbf{F} \times \boldsymbol{d}$). The distance $\boldsymbol{d}$ is termed the moment arm.
3. **Newton's third law:** if a body A exerts a force on a body B, then body B exerts an equal and oppositely directed force on A.
4. **Balancing forces and moments:** in two dimensions (2D) forces can be balanced in two mutually perpendicular directions, vertically and horizontally; moments are balanced about a point in clockwise and anti-clockwise directions. The convention is for clockwise moments to act in a positive sense.

We now turn to our biomechanics problem solved with rigid body statics: the use of a walking stick in the correct hand for hip arthritis. The analysis assumes that: the weight of the upper body acts through the centre of the pelvis; all the body segments are rigid bodies; only the frontal plane is considered (2D); only the abductor muscles are considered to act.

Consider a person of mass $\boldsymbol{m}$. The weight of the person is $\boldsymbol{mg}$ (mass × acceleration due to gravity). The weight of each leg is 15% of body weight, or 0.15 $\boldsymbol{mg}$, and therefore the weight of the upper body (head, arms and trunk) is 0.7 $\boldsymbol{mg}$. Figure 1a shows the forces acting on the pelvis and femora when the person is standing on two legs and bearing equal weight on each leg). The joint reaction force, ***JRF***, acts at the hip joint centre; the femur applies a force to the pelvis and the pelvis applies an equal and opposite force on the femur (Newton's third law). The abductor muscle can only act in tension and applies a force $\boldsymbol{A}$ at its attachments to the pelvis and the femur (again equal and opposite due to Newton's third law). There will be two reactions from the ground (not shown in Figure 1a) that act at each of the feet, equal and opposite to the weight of the upper body.

Figure 1b is a free body diagram of the pelvis; the other bodies (the femora) are not shown, but their effects are represented by the forces acting on the pelvis. The abductor force $\boldsymbol{A}$ acts at an angle α to the vertical and the ***JRF*** acts at an angle β to the vertical. These forces can be resolved into the horizontal and vertical direction, which is shown in the free body diagram in Figure 1c.

We will now consider three cases: 1) the person standing on one leg, and ignoring the effects of the other leg; 2) the person standing on one leg and including the other leg; 3) the person

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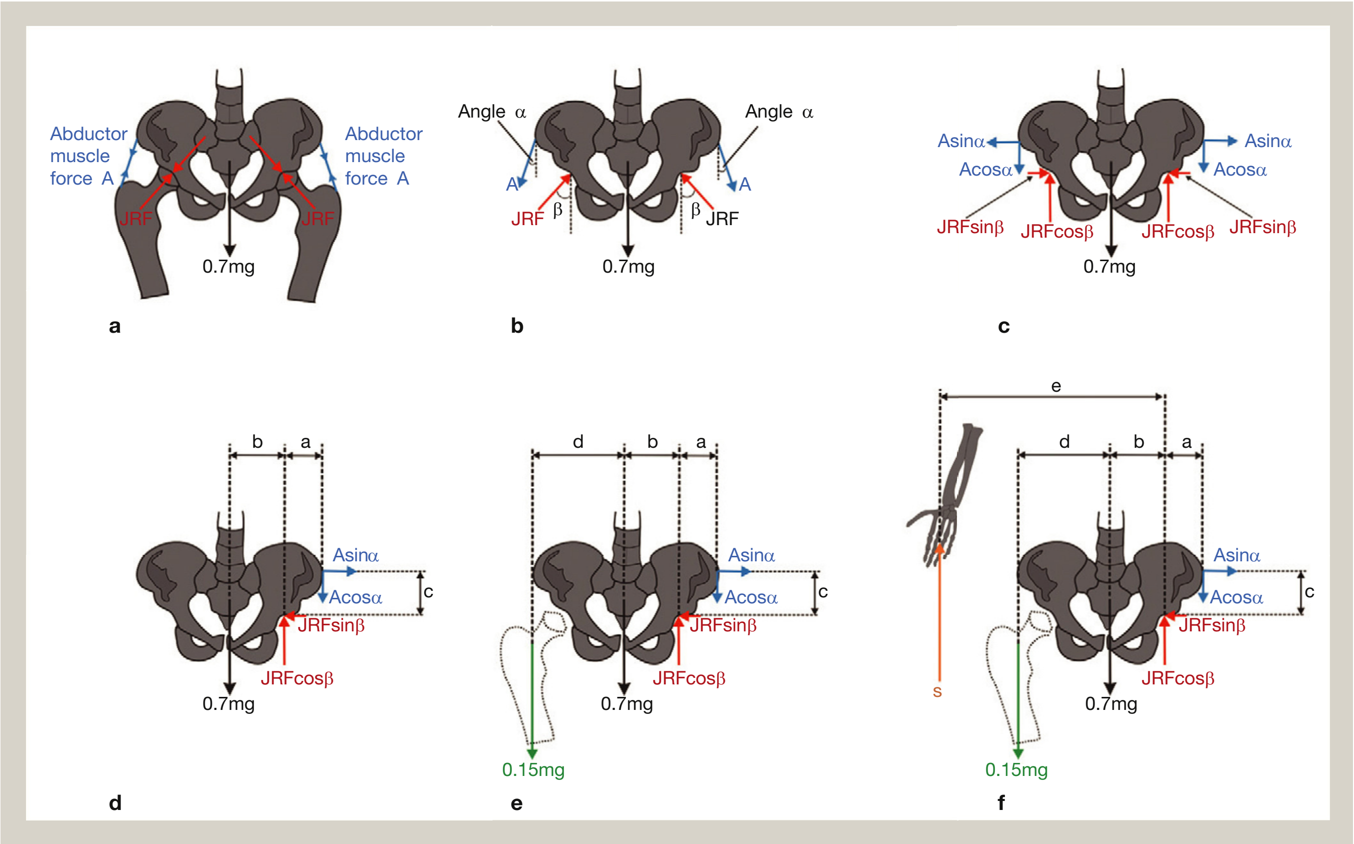

Figure 1 (**a**) Forces acting on the pelvis and femur for a person standing on two legs. (**b**) Free body diagram of the pelvis whilst standing on two legs. (**c**) Free body diagram of the pelvis whilst standing on two legs with forces resolved in vertical and horizontal directions. (**d**) Free body diagram of the pelvis whilst standing on one leg (left) and ignoring the right leg. (**e**) Free body diagram of the pelvis whilst standing on one leg (left) and including the right leg force (0.15 *mg*). (**f**) Assistance of a walking stick (S) with an arthritic left hip.

standing on one leg, including the other leg, and using a walking stick.

1. Standing on one leg and ignoring the other leg

Figure 1d shows that whilst the person stands on the left leg there will be no joint reaction force acting at the right hip, and there will be no muscle activity in the right abductors. The force balance is:

In the vertical direction

$$JRF \times \cos(\beta) = 0.7 \times mg + A \times \cos(\alpha)$$

$$JRF\cos(\beta) = 0.7\ mg + A\cos(\alpha)$$

In the horizontal direction

$$JRF\sin(\beta) = A\sin(\alpha)$$

$$JRF = \frac{A\sin(\alpha)}{\sin(\beta)}$$

The moment balance about centre of left hip (note that as *JRF* acts through the centre of the left hip, it does not generate a moment about it because its moment arm is zero):

$$c \times A\sin(\alpha) + a \times A\cos(\alpha) = 0.7\ mg \times b$$
$$A[c\sin(\alpha) + a\cos(\alpha)] = 0.7\ mgb$$

$$A = mg\left(\frac{0.7b}{c\sin(\alpha) + a\cos(\alpha)}\right)$$

Let us consider the expression for the abductor muscle force, ***A***, and ***JRF*** with some numerical data:

$a = 120$ mm, $b = 270$ mm, $c = 105$ mm, $\alpha = 5°$ $\beta = 3°$.

With these numerical values the abductor force, **A,** is equal to 1.5 ***mg***, or one and a half times body weight. The ***JRF*** is 2.4 ***mg***, or approximately two and a half times body weight.

It is important to note that in the human body internally applied loads (muscle forces) are associated with small moment arms and externally applied loads (due to gravity) are associated with large lever arms. Therefore the internally applied loads (such as the abductor muscle and ***JRF***) are considerably larger than the externally applied loads (such as body weight).

2. Standing on one leg and including the other leg

Now consider the person standing on one leg, but with the inclusion of the hanging leg (Figure 1e). The weight of the free leg (0.15 ***mg***) is considered to act at a perpendicular distance of **(*d* + *b*)** from the left hip joint centre. Note that in this example