Expert Systems With Applications 52 (2016) 1–15

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

Self-evolution of hyper fractional order chaos driven by a novel approach through genetic programming☆

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A R T I C L E I N F O

MSC:
90C59
90C56
90C30

Keywords:
Fractional-order chaos
Self-evolution
United functional extrema model
Genetic programming

A B S T R A C T

To find best inherent chaotic systems behind the complex phenomena is of vital important in Complexity science research. In this paper, a novel non-Lyapunov methodology is proposed to self-evolve the best hyper fractional order chaos automatically driven by a computational intelligent method, genetic programming. Rather than the unknown systematic parameters and fractional orders, the expressions of fractional-order differential equations (FODE) are taken as particular independent variables of a proper converted non-negative minimization of special functional extrema in the proposed united functional extrema model (UFEM), then it is free of the hypotheses that the definite forms of FODE are given but some parameters and fractional orders unknown. To demonstrate the potential of the proposed methodology, simulations are done to evolve a series of benchmark hyper and normal fractional chaotic systems in complexity science. The experiments' results show that the proposed paradigm of fractional order chaos driven by genetic programming is a successful method for chaos' automatic self-evolution, with the advantages of high precision and robustness.

1. Introduction

Complex systems are ubiquitous in physics, sociology, biology, economics and many other scientific areas (Holland, 1998). Normally, a complex system consists of tiny aggregated components, whose interaction and interconnectedness are nontrivial anyway. Meanwhile, the interactions result in high-dimensional, nonlinear interactions and connectivity leads to nontrivial topological features such as high clustering coefficients and power-law degree distribution. These bring about emergent properties of the complex system, not anticipated by its own isolated components. Furthermore, when system behaviour is studied form a temporal perspective, self-organisation patterns come into being typically. After the symbolic event that Mandelbort discovered that there were a lot of fractional dimension phenomena in nature (Mandelbrot, 1982) in bifurcation, hyperchaos, proper and improper fractional-order chaos systems and chaos synchronization (Diethelm & Ford, 2002; Doha, Bhrawy, & Ezz-Eldien, 2012; Odibat, Corson, Aziz-Alaoui, & Bertelle, 2010; Si, Sun, Zhang, & Chen, 2012b; Song, Yang, & Xu, 2010; Tavazoei & Haeri, 2007; 2008), the applications of fractional differential equations began to appeal to related scientists (Agrawal, Srivastava, & Das, 2012; Bhalekar & Daftardar-Gejji, 2010; Diethelm, Ford, & Freed, 2002; Grigorenko & Grigorenko, 2003; Kaslik & Sivasundaram, 2012; Odibat, 2012; Odibat et al., 2010; Tang, Zhang, Hua, Li, & Yang, 2012; Tavazoei & Haeri, 2007).

Studying complex fractional order systems requires composite strategies, which employ diverse algorithms to solve a single but difficult problem. Components of these strategies may resolve consecutive phases leading to the final goal, or may resolve the main problem in different methods which can be aggregated to form the final solution. For example, the hyper-heuristics, evolutionary

☆ The work was carried out during the tenure of the ERCIM Alain Bensoussan Fellowship Programme, which is supported by the Marie Curie Co-funding of Regional, National and International Programmes (COFUND) of the European Commission. The work is partially supported by Supported by the State Key Program of National Natural Science of China (Grant No. 91324201), the National Natural Science Foundation of China (Grant No. 81271513) of China, the Fundamental Research Funds for the Central Universities of China, the self–determined and innovative research funds of WUT No. 2014-Ia-012, 2013-Ia-040, the Natural Science Foundation No. 2014CFB865 of Hubei Province of China, Scientific Research Foundation for Returned Scholars from Ministry of Education of China (No. 20111j0032), the HOME Program No. 11044 (Help Our Motherland through Elite Intellectual Resources from Overseas) funded by China Association for Science and Technology,The National Soft Science Research Program 2009GXS1D012 of China, the National Science Foundation for Post–doctoral Scientists of China No. 20080431004.

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http://dx.doi.org/10.1016/j.eswa.2015.12.033
0957-4174/

algorithms (Gao et al., 2014a; 2014b), island Genetic algorithms (GA), swarm intelligent methods (Gao, Lee, Tong, Fei, & Zhao, 2013; Gao, Qi, Yin, & Xiao, 2010a; 2010b; Gao & Tong, 2006) and the other approaches are often used.

The paper will proposed a novel approach to simulate the evolution process driven by Genetic Programming (GP) of the fractional order complex systems in an generalized identification perspective. It is difficult to identify the expressions or parameters in the uncertain fractional-order chaotic systems. There are some systematic parameters and orders are unknown for the fractional-order chaos systems in controlling and synchronization.

Hitherto, there have been at two main approaches in generalized parameters' identification for fractional-order chaos systems. The first is a Lyapunov way using synchronisation. And there have been few results on parameter estimation method of fractional-order chaotic systems based on chaos synchronization (Si, Sun, Zhang, & Zhang, 2012a). However, the design of controller and the updating law of parameter identification is a task with technique and sensitively depends on the considered systems. The second is a non-Lyapunov way using some artificial intelligence methods, such as differential evolution (Tang et al., 2012) and particle swarm optimization (Yuan & Yang, 2012).

Although there are many methods in parameters estimation for integer order chaos systems (Alonso et al., 2015; Chang, Yang, Liao, & Yan, 2008a; Fan, Li, & Tian, 2015; Gao, xia Fei, fang Deng, bo Qi, & Ilangko, 2012a; Gao, Qi, Balasingham, Yin, & Gao, 2012b; Mukhopadhyay & Banerjee, 2012; Pan & Das, 2015; dos Santos, Luvizotto, Mariani, & dos Santos Coelho, 2012; Wang, Xu, & Li, 2011) , to the best of our knowledge, a little work in non-Lyapunov way has been done to the unknown expression or the parameters and orders estimation of fractional-order chaos systems (Gao et al., 2014a; 2014b; Gao et al., 2013; Tang et al., 2012; Yuan & Yang, 2012).

For example, we consider the following fractional-order chaos system.

$$ {}_{\alpha}\mathscr{D}_t^q Y(t) = f(Y(t), Y_0(t), \theta) \tag{1}$$

where $Y(t) = (y_1(t), y_2(t), \ldots, y_n(t))^T \in \Re^n$ denotes the state vector. $\theta = (\theta_1, \theta_2, \ldots, \theta_n)^T$ denotes the original parameters. $q = (q_1, q_2, \ldots, q_n)$, $(0 < q_i < 1, i = 1, 2, \ldots, n)$ is the fractional derivative orders.

$$ f(Y(t), Y_0(t), \theta) = (f_1, f_2, \ldots, f_n)|_{(Y(t), Y_0(t), \theta)} $$

The continuous integro-differential operator (Petráš, 2011a; 2011b) is defined as

$$ {}_{\alpha}\mathscr{D}y_t^q = \begin{cases} \frac{d^q}{dx^q}, q > 0; \\ 1, q = 0; \\ \int_{\alpha}^{1} (d\tau)^q. \end{cases} $$

Normally the function expression f in Eq. (1) is known. But some the θ, q are unknown , then the $\Theta = (\theta_1, \theta_2, \ldots, \theta_n, q_1, q_2, \ldots, q_n)$ will be the parameters to be estimated, through a constructed evaluation function F. And the objective is obtained as following Eq. (2),

$$ \theta^* = \arg\min_{\theta} F \tag{2}$$

However, there exist basic hypotheses in traditional non-Lyapunov estimation methods (Al-Assaf, El-Khazali, & Ahmad, 2004; Tang et al., 2012; Yuan & Yang, 2012) on generalized parameters' identification. That is, the parameters and fractional orders $\Theta = (\theta_1, \theta_2, \ldots, \theta_n, q_1, q_2, \ldots, q_n)$ are partially unknown to be estimated. But few works are valid in the case the definite expression forms $f = (f_1, f_2, \ldots, f_n)$. What should I do when these hypotheses do not exist? That is, when some the fractional order differential equations (FODE)' expression $f = (f_1, f_2, \ldots, f_n)$ are unknown, how to identify the fractional system in general? That is,

$$ (f_1, f_2, \ldots, f_n)^* = \arg\min_{(f_1, f_2, \ldots, f_n)} F \tag{3}$$

Now the problem of parameters estimation (2) become another much more complicated question, to find the forms of fractional order equations as in (3). In another word, it is fractional-order chaos' evolution driven by some methods in reconstruction perspective now.

However, to the best of authors' knowledge, there are no methods in non-Lyapunov way for fractional order chaotic systems' reconstruction for system evolution so far. The objective of this work is to present a novel simple but effective approach to evolve the hyper, proper and improper fractional chaotic systems in a non-Lyapunov way. In which, an idea of self-evolved fractional order chaos driven by genetic programming are used to identify unknown expressions' forms of FODE $f = (f_1, f_2, \ldots, f_n)$, in which no extra evolutionary algorithms to be added or be compounded. And the illustrative evolution simulations for diverse chaos systems system are discussed respectively.

The rest is organized as follows. Section 2 gives a simple review on non-Lyapunove parameters estimation methods for fractional-order chaos systems. In Section 3, a novel united mathematical model for fractional chaos evolution are proposed in a non-Lyapunov way. Then all the existing non-Lyapunov methods in Section 2 become special cases of the new united model, such as non-Lyapunov parameters identification for normal and fractional chaos systems, non-lyapunov reconstruction methods for normal chaos system. In Section 3.3 a novel methods with proposed united model in Section 3 is introduced to reconstruct the hyper, improper and normal fractional chaos systems. And simulations are done to a series of different fractional chaos systems and their special cases in three dimension. Conclusions are summarized briefly in Section 5.

2. Non-Lyapunove reconstruction methods for normal chaos systems

To solve the question proposed in Section 1, a new ideas are introduced to generalize the normal parameters' estimation problems into fractional order chaos reconstruction problems. In the sense of the differential equations for chaos system reconstruction, the parameters' identification for normal chaos system cases (Chang, Yang, Liao, & Yan, 2008b; Chang, 2007; Gao, Lee, Li, Tong, & Lü, 2009b; Gao, Li, & Tong, 2008; Gao et al., 2012b; Gao, Qi, Yin, & Xiao, 2012c; Gao & Tong, 2006; Guan, Peng, Li, & Wang, 2001; Li, Yang, Peng, & Wang, 2006; Parlitz, 1996; Shen & Wang, 2008; Yang, Maginu, & Nomura, 2009) and for the fractional Lorenz system (Deng & Li, 2005; Grigorenko & Grigorenko, 2003; Lu, 2006; Wu & Shen, 2009; Yu, Li, Wang, & Yu, 2009) can be thought as special cases of fractional order chaos reconstruction, when the exact forms of fractional order chaotic differential equations $f = (f_1, f_2, \ldots, f_n)$ are available but some parameters unknown.

To propose a novel reconstruction way for fractional-order chaos system, we briefly reviewed the normal chaos reconstruction methods as following. Firstly, we introduce the basic artificial intelligent symbolic regression method Genetic programming (GP) (Koza, 1992; 1994; Pan & Das, 2015; Poli, Langdon, & McPhee, 2008; Vyas, Goel, & Tambe, 2015). Secondly, the non-Lyapunov way for normal chaos reconstruction are also introduced.

2.1. Genetic programming

To have a deep understanding of reconstruction method in non-Lyapunov way, firstly we introduce some basic ideas of GP.