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A fuzzy system for evaluating students' learning achievement

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ABSTRACT

The proper system for evaluating the learning achievement of students is the key to realizing the purpose of education. In recent years, several methods have been presented for applying the fuzzy set theory in the educational grading systems. In this paper, we propose a method for the evaluation of students' answerscripts using a fuzzy system. The proposed system applies fuzzification, fuzzy inference, and defuzzification in considering the difficulty, the importance and the complexity of questions. The transparency, objectivity, and easy implementation of the proposed fuzzy system provide a useful way to automatically evaluate students' achievement in a more reasonable and fairer manner.

1. Introduction

Evaluation of students' learning achievement is the process of determining the performance levels of individual students in relation to educational objectives. A high quality evaluation system certifies, provides grounds for individual improvement, and ensures that all students receive fair grading so as not to limit students' present and future opportunities. Thus, the system should regularly be reviewed and improved to ensure that it is precise, fair, and beneficial to all students. Hence, the evaluation system needs the transparency, objectivity, logical reasoning, and easy computer implementation which could be provided by the fuzzy logic system.

Since its introduction in 1965 by Zadeh (1965), the fuzzy set theory has been widely used in solving problems in various fields, and recently in educational grading systems. Biswas (1995) presented two methods for the evaluation of students' answerscripts using fuzzy sets and a matching function: a fuzzy evaluation method and a generalized fuzzy evaluation method. Chen and Lee (1999) presented two methods for applying fuzzy sets to overcome the problem of giving two different fuzzy marks to students with the same total score which could arise from Biswas' method. Echauz and Vachtsevanos (1995) proposed a fuzzy logic system for translating traditional scores into letter-grades. Law (1996) built a fuzzy structure model for an educational grading system with its algorithm to aggregate different test scores in order to produce a single score for individual students. He also proposed a method to build the membership functions (MFs) of several linguistic values with different weights. Wilson, Karr, and Freeman (1998) presented an automatic grading system based on fuzzy rules and genetic algorithms. Ma and Zhou (2000) proposed a fuzzy set approach to assess the outcomes of student-centered learning using the evaluation of their peers and lecturer. Wang and Chen (2008) presented a method for evaluating students' answerscripts using fuzzy numbers associated with degrees of confidence of the evaluator. From the previous studies, it can be found that fuzzy numbers, fuzzy sets, fuzzy rules, and fuzzy logic systems have been used for various educational grading systems.

Weon and Kim (2001) developed an evaluation strategy based on fuzzy MFs. They pointed out that the system for students' achievement evaluation should consider the three important factors of the questions given to students: the difficulty, the importance, and the complexity. Weon and Kim used singleton functions to describe the factors of each question reflecting the individual effect of the three factors, but not the collective effect. Bai and Chen (2008b) pointed out that the difficulty factor is a very subjective parameter and may cause an argument concerning fairness in evaluation.

Bai and Chen (2008a) proposed a method to automatically construct the grade MFs of fuzzy rules for evaluating student's learning achievement. Bai and Chen (2008b) proposed a method for applying fuzzy MFs and fuzzy rules for the same purpose. To solve the subjectivity of the difficulty factor in Weon and Kim's method (2001), they obtained the level of difficulty as a function of the accuracy of the student's answerscript and the time consumed to answer the questions. However, their method still has the subjectivity problem, since the results in scores and ranks are heavily dependent on the values of several weights which are determined by the subjective knowledge of domain experts.

In this paper, as an improved alternative to Bai and Chen's method (2008b), we propose a fuzzy logic evaluation system considering the importance, the difficulty, and the complexity of questions based on Mamdani's fuzzy inference (Mamdani, 1974) and

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center of gravity (COG) defuzzification. The transparency and objective nature of the fuzzy logic system make it easy to understand and explain the results of evaluation, and thus to persuade students who are skeptical or not satisfied with the evaluation results.

The paper is organized as follows. In Section 2, we consider the same structure of evaluating students' learning achievement as Bai and Chen's (2008b) and introduce their solution method using fuzzy MFs and fuzzy rules. In Section 3, we propose a three node fuzzy evaluation system. The procedure consists of fuzzification, inference, and defuzzification. In Section 4, through an example, the procedure of the proposed system is explained and its result is compared with Bai and Chen's. Conclusions are drawn in Section 5.

2. A review of evaluation methods using membership functions and fuzzy rules

In this paper, we consider the same situation and example as in Bai and Chen's (2008b). Assume that there are n students to answer m questions. Accuracy rates of students' answerscripts (student's scores in each question divided by the maximum score assigned to this question) are the basis for evaluation. We get an accuracy rate matrix of dimension $m \times n$,

$$A = [a_{ij}], \quad m \times n,$$

where $a_{ij} \in [0,1]$ denotes the accuracy rate of student j on question i. Time rates of students (time consumed by a student to solve a question divided by the maximum time allowed to solve this question) is another basis to be considered in the evaluation. We get a time rate matrix of dimension $m \times n$,

$$T = [t_{ij}], \quad m \times n,$$

where $t_{ij} \in [0,1]$ denotes the time rate of student j on question i. We are given a grade vector

$$G = [g_i], \quad m \times 1,$$

where $g_i \in [1,100]$ denotes the assigned maximum score of question i satisfying

$$\sum_{i=1}^{m} g_i = 100.$$

Based on the accuracy rate matrix A and the grade vector G, we obtain the original total score vector of dimension $n \times 1$,

$$S = A^{\mathrm{T}} G = [s_j], \quad n \times 1, \tag{1}$$

where $s_j \in [0,100]$ is the total score of student j. The "classical" ranks of students are then obtained by sorting the element values of S in descending order.

Example. Assume that 10 students are given an examination of 5 questions and the accuracy rate matrix, the time rate matrix, and the grade vector are given as follows (Bai & Chen, 2008b):

$$A = \begin{bmatrix} 0.59 & 0.35 & 1 & 0.66 & 0.11 & 0.08 & 0.84 & 0.23 & 0.04 & 0.24 \\ 0.01 & 0.27 & 0.14 & 0.04 & 0.88 & 0.16 & 0.04 & 0.22 & 0.81 & 0.53 \\ 0.77 & 0.69 & 0.97 & 0.71 & 0.17 & 0.86 & 0.87 & 0.42 & 0.91 & 0.74 \\ 0.73 & 0.72 & 0.18 & 0.16 & 0.5 & 0.02 & 0.32 & 0.92 & 0.9 & 0.25 \\ 0.93 & 0.49 & 0.08 & 0.81 & 0.65 & 0.93 & 0.39 & 0.51 & 0.97 & 0.61 \end{bmatrix},$$

$$T = \begin{bmatrix} 0.7 & 0.4 & 0.1 & 1 & 0.7 & 0.2 & 0.7 & 0.6 & 0.4 & 0.9 \\ 1 & 0 & 0.9 & 0.3 & 1 & 0.3 & 0.2 & 0.8 & 0 & 0.3 \\ 0 & 0.1 & 0 & 0.1 & 0.9 & 1 & 0.2 & 0.3 & 0.1 & 0.4 \\ 0.2 & 0.1 & 0 & 1 & 1 & 0.3 & 0.4 & 0.8 & 0.7 & 0.5 \\ 0 & 0.1 & 1 & 1 & 0.6 & 1 & 0.8 & 0.2 & 0.8 & 0.2 \end{bmatrix},$$

$$G^{\mathrm{T}} = [10 \quad 15 \quad 20 \quad 25 \quad 30].$$

In this paper, V^{T} denotes the transpose of vector V. □

The importance of the questions is an important factor to be considered. We have l levels of importance to describe the degree of importance of each question in the fuzzy domain. The domain expert determines the importance matrix of dimension $m \times l$

$$P = [p_{ik}], \quad m \times l,$$

where $p_{ik} \in [0,1]$ denotes the membership value (degree of the membership) of question i belonging to the importance level k. In this paper, five levels (fuzzy sets) of importance ($l = 5$) are used; $k = 1$ for linguistic term "low", $k = 2$ for "more or less low", $k = 3$ "medium", $k = 4$ for "more or less high", and $k = 5$ for "high". Their MFs are shown in Fig. 1. We note that the same five fuzzy sets are applied to the accuracy, the time rate, the difficulty, the complexity, and the adjustment of questions. Once crisp values are given for the importance of questions by a domain expert, the values of p_{ik}'s are obtained by the fuzzification.

The complexity of the questions which indicates the ability of students to give correct answers is also an important factor to be considered. The domain expert determines the fuzzy complexity matrix of dimension $m \times l$,

$$C = [c_{ik}], \quad m \times l,$$

where $c_{ik} \in [0,1]$ denotes the membership value of question i belonging to the complexity level k.

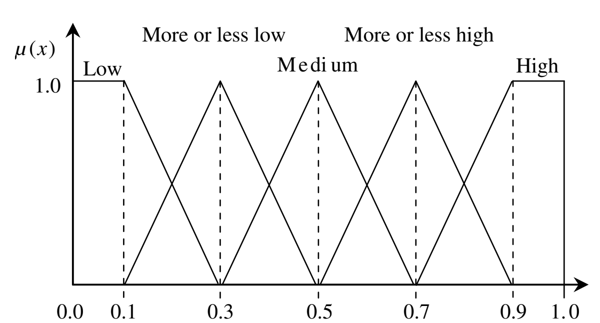

Fig. 1. Fuzzy membership functions of the five levels.