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Forecasting in high order fuzzy times series by using neural networks to define fuzzy relations

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ABSTRACT

A given observation in time series does not only depend on preceding one but also previous ones in general. Therefore, high order fuzzy time series approach might obtain better forecasts than does first order fuzzy time series approach. Defining fuzzy relation in high order fuzzy time series approach are more complicated than that in first order fuzzy time series approach. A new proposed approach, which uses feed forward neural networks to define fuzzy relation in high order fuzzy time series, is introduced in this paper. The new proposed approach is applied to well-known enrollment data for the University of Alabama and obtained results are compared with other methods proposed in the literature. It is found that the proposed method produces better forecasts than the other methods.

1. Introduction

In recent years, fuzzy time series approach introduced by Song and Chissom (1993a, 1993b) has been used widely. Chen (1996) proposed a method which is simpler than the method proposed by Song and Chissom (1993a, 1993b) in forecasting fuzzy time series. The method proposed by Chen (1996) does not include complex matrix operations in defining fuzzy relation. Huarng and Hui-Kuang (2006) uses a simple feed forward neural network to define fuzzy relation. Due to the first order fuzzy times series approach implementation in Huarng and Hui-Kuang (2006), his proposed method includes a simple feed forward neural network model which has one input neuron, two hidden layers' neurons, and one output neuron. Because of not losing the generalization ability of neural network model, Hurang used two neurons in hidden layer.

Hwang, Chen, and Lee (1998) and Chen (2002) used high order fuzzy time series model. Chen's (2002) model consists of defining fuzzy relation based on previous observations. The implementation of Chen's approach becomes more difficult when the order of fuzzy time series increases. However, neural networks can be used easily for high order fuzzy time series. In this study, feed forward neural networks are employed to define fuzzy relation by trying various architectures for high order fuzzy time series. The proposed approach based on neural networks is applied to well-known enrollment data for University of Alabama. Obtained results are compared with other methods and it is clearly seen that our proposed method has better forecasting accuracy when compared with other methods proposed in the literature.

Section 2 includes the definitions of first and high order time series. The brief information related to neural networks is given in Section 3. The new proposed method is introduced and the implementation results of enrollment data are given in Sections 4 and 5 respectively. Final section is for conclusion.

2. Fuzzy time series

The definition of fuzzy time series was firstly introduced by Song and Chissom (1993a, 1993b). In fuzzy time series approximation, you do not need various theoretical assumptions just as you need in conventional time series procedures. The most important advantage of fuzzy time series approximations is to be able to work with a very small set of data and not to require the linearity assumption. The some general definitions of fuzzy time series are given as follows:

Let U be the universe of discourse, where $U = \{u_1, u_2, \ldots, u_b\}$. A fuzzy set A_i of U is defined as $A_i = f_{A_i}(u_1)/u_1 + f_{A_i}(u_2)/u_2 + \cdots + f_{A_i}(u_b)/u_b$, where f_{A_i} is the membership function of the fuzzy set $A_i; f_{A_i} : U \to [0,1]$. u_a is a generic element of fuzzy set $A_i; f_{A_i}(u_a)$ is the degree of belongingness of u_a to $A_i; f_{A_i}(u_a) \in [0,1]$ and $1 \leqslant a \leqslant b$.

Definition 1. Fuzzy time series Let $Y(t)(t = \ldots, 0, 1, 2, \ldots)$ a subset of real numbers, be the universe of discourse by which fuzzy sets $f_j(t)$ are defined. If $F(t)$ is a collection of $f_1(t), f_2(t), \ldots$ then $F(t)$ is called a fuzzy time series defined on $Y(t)$.

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Definition 2. Fuzzy time series relationships assume that $F(t)$ is caused only by $F(t-1)$, then the relationship can be expressed as: $F(t) = F(t-1)^{*}R(t,t-1)$, which is the fuzzy relationship between $F(t)$ and $F(t-1)$, where * represents as an operator. To sum up, let $F(t-1) = A_i$ and $F(t) = A_j$. The fuzzy logical relationship between $F(t)$ and $F(t-1)$ can be denoted as $A_i \rightarrow A_j$ where A_i refers to the left-hand side and A_j refers to the right-hand side of the fuzzy logical relationship. Furthermore, these fuzzy logical relationships can be grouped to establish different fuzzy relationship.

Definition 3. Let $F(t)$ be a fuzzy time series. If $F(t)$ is a caused by $F(t-1)$,$F(t-2)$,...,$F(t-m)$, then this fuzzy logical relationship is represented by

$$F(t-m), \ldots, F(t-2),\ F(t-1) \rightarrow F(t),$$

and it is called the mth order fuzzy time series forecasting model.

3. Artificial neural networks

'What is an artificial neural network?' is the first question that should be answered. Picton (1994) answered this question by separating this question into two parts. The first part is why it is called an artificial neural network. It is called an artificial neural network because it is a network of interconnected elements. These elements were inspired from studies of biological nervous systems. In other words, artificial neural networks are an attempt at creating machines that work in a similar way to the human brain by building these machines using components that behave like biological neurons. The second part is what an artificial neural network does. The function of an artificial neural network is to produce an output pattern when presented with an input pattern. In forecasting, artificial neural networks are mathematical models that imitate biological neural networks. Artificial neural networks consist of some elements. Determining the elements of the artificial neural networks issue that affect the forecasting performance of artificial neural networks should be considered carefully. Elements of the artificial neural networks are generally given as network architecture, learning algorithm and activation function. One critical decision is to determine the appropriate architecture, that is, the number of layers, number of nodes in each layers and the number of arcs which interconnects with the nodes (Zurada, 1992). However, in the literature, there are not general rules for determining the best architecture. Therefore, many architecture should be tried for the correct results. There are various types of artificial neural networks. One of them is called as feed forward neural networks. The feed forward neural networks have been used successfully in many studies. In the feed forward neural networks, there are no feedback connections. Fig. 1 depicts the broad feed forward neural network architecture that has single hidden layer and single output.

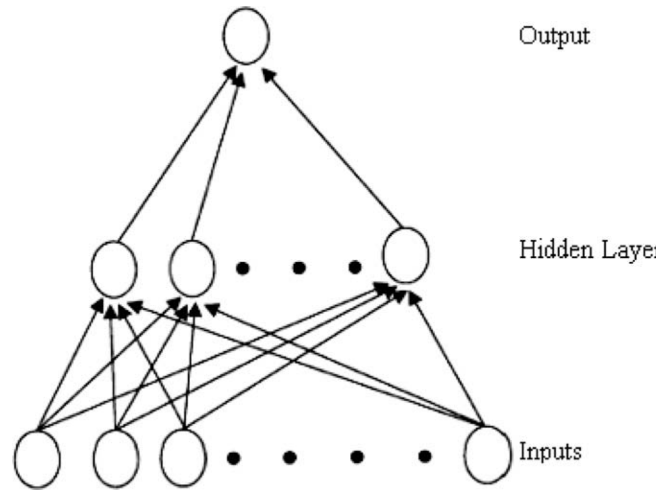

Fig. 1. A broad feed forward neural network architecture.

Learning of an artificial neural network for a specific task is equivalent to finding the values of all weights such that the desired output is generated by the corresponding input. Various training algorithms have been used for the determination of the optimal weights values. The most popularly used training method is the back propagation algorithm presented by Smith (2002). In the back propagation algorithm, learning of the artificial neural networks consists of adjusting all weights considering the error measure between the desired output and actual output (Cichocki & Unbehauen, 1993). Another element of the artificial neural networks is the activation function. It determines the relationship between inputs and outputs of a network. In general, the activation function introduces a degree of the non-linearity that is valuable in most of the artificial neural networks applications. The well-known activation functions are logistic, hyperbolic tangent, sine (or cosine) and the linear functions. Among them, logistic activation function is the most popular one (Zhang, Patuwo, & Hu, 1998).

In the application, feed forward neural networks architecture, which includes one hidden layer and one output, is used to define fuzzy relation. Back propagation learning algorithm is used to train neural network models and logistic activation function is employed in all neurons.

4. The proposed method

In order to construct high order fuzzy time series model, various feed forward neural networks architectures are employed to define fuzzy relation. The stages of the proposed method based on neural networks are given below.

Stage 1. Define and partition the universe of discourse
The universe of discourse for observations, U = [starting, ending], is defined. After the length of intervals, l, is determined, the U can be partitioned into equal-length intervals u_1, u_2,...,u_b, $b = 1,\ldots$ and their corresponding midpoints m_1,m_2,...,m_b, respectively.

$$u_b = [\text{starting} + (b-1) \times l, \text{starting} + b \times l],$$
$$m_b = \frac{[\text{starting} + (b-1) \times l, \text{starting} + b \times l]}{2}$$

Stage 2. Define fuzzy sets.
Each linguistic observation, A_i, can be defined by the intervals u_1, u_2,...,u_b.

$$A_i = f_{A_i}(u_1)/u_1 + f_{A_i}(u_2)/u_2 + \cdots + f_{A_i}(u_b)/u_b$$

Stage 3. Fuzzify the observations.
For example, a datum is fuzzified to A_i, if the maximal degree of membership of that datum is in A_i.

Stage 4. Establish the fuzzy relationship with feed forward neural network.
An example will be given to explain stage 4 more clearly for the second order fuzzy time series. Because of dealing with second order fuzzy time series, two inputs are employed in neural network model, so that lagged variables F_{t-2} and F_{t-1} are obtained from fuzzy time series F_t. These series are given in Table 1. The index numbers (i) of A_i of F_{t-2} and F_{t-1} series are taken as input values whose titles are Input-1 and Input-2 in Table 1 for the neural network model. Also, the index numbers of A_i of