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A generalized model for prioritized multicriteria decision making systems

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ABSTRACT

In this paper, we present a generalized model for handling prioritized multicriteria decision making systems. First, we present a new method for handling prioritized multicriteria decision making problems, where the weights of the lower priority criteria of each alternative depend on whether each alternative satisfies the requirements of all the higher priority criteria or not. Then, we also present a generalized prioritized multicriteria decision making method for handling multicriteria decision making problems, where some criteria may have equal priority which can be aggregated by the ordered weighted averaging (OWA) operator or the weighted averaging method. The proposed methods can overcome the drawbacks of the methods presented in [Yager, R. R. (2004a). Modeling prioritized multicriteria decision making. *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics*, 34(6), 2396–2404]. The proposed methods can handle multicriteria decision making problems in a more intelligent and more flexible manner.

1. Introduction

In recent years, some methods have been presented for handling multicriteria decision making problems (Bellman & Zadeh, 1970; Beynon, Cosker, & Marshall, 2001; Bordogna & Pasi, 1995; Chen, 1988; Chen & Chen, 2005; Fu, 2008; Filev & Yager (1995, 1999a, 1999b); Fullér & Majlender, 2003; Kulak, 2005; Kwon, Kim, & Lee, 2007; Kwon & Kim, 2004; Lin, Hsu, & Sheen, 2007; Liu, 2006; Tacker & Silvia, 1991; Torra, 1997; Wang & Chen, 2007; Yager (1988, 1991, 1992, 1996, 1998, 2004a, 2004b)). Some methods have been presented for handling prioritized multicriteria decision making problems (Chen & Chen, 2005; Wang & Chen, 2007; Yager (1991, 1992, 1998, 2004b)). Yager (1991) presented a prioritized intersection operator, called the non-monotonic intersection operator, for default and other common-sense reasoning systems. Yager (1992) used the non-monotonic intersection operator to deal with multicriteria decision making problems and presented a type of criterion, called the second order criterion. A statement shown in (Yager, 1992), such as "I want a good job, *near my house if possible*", involves the first criterion (i.e., a good job) and the second criterion (i.e., *near my house if possible*). A second order criterion acts as an additional selector on these alternatives which satisfy the first order criteria. If none of the elements which satisfy the first order criteria also satisfy the second order criteria, then we do not need to consider the requirements of the second order criteria. Bordogna and Pasi (1995) regarded the process of information retrieval as a multicriteria decision making activity. They used the prioritized intersection operator to deal with fuzzy information retrieval problems. Yager (1998) used the weighted conjunction of fuzzy sets and fuzzy modeling to develop the operators in fuzzy information fusion structures. Chen and Chen (2005) extended the non-monotonic intersection operator (Yager, 1991) to present a prioritized information fusion method for handling prioritized multicriteria fuzzy decision making problems based on the similarity measure of generalized fuzzy numbers. Yager (2004a) presented some methods for handling prioritized multicriteria decision making problems, based on the Bellman–Zadeh paradigm for multicriteria decision making and the ordered weighted averaging operator (OWA). In prioritized multicriteria decision making problems, some criteria may be necessary to be satisfied. An example shown in Yager (2004a) is the case of air travel, where the passengers' safety is necessary to be satisfied. In this case, the passengers' safety has a higher priority than saving gasoline. Tradeoffs between saving gasoline and jeopardizing the passengers' safety are unacceptable. Yager (2004a) found that the weights associated with the lower priority criteria are related to the degree of satisfaction of the alternatives with respect to the higher priority criteria.

In this paper, we present a generalized model for prioritized multicriteria decision making systems. First, we present a new method for prioritized multicriteria decision making, where the weights of the lower priority criteria of each alternative depend on whether each alternative satisfies the requirements of all the higher priority criteria or not. If the requirements of all the higher priority criteria can not be satisfied by the alternative, then the

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weights of the lower priority criteria are all zero. That is, the degrees of satisfaction with respect to the lower priority criteria do not affect the overall degree of satisfaction. Then, we present a generalized prioritized multicriteria decision making method for handling multicriteria decision making problems in which some criteria may have equal priority and the criteria with equal priority are aggregated by using the ordered weighted averaging (OWA) operator or the weighted averaging method. The proposed methods can overcome the drawbacks of the methods presented in (Yager, 2004a). They can handle multicriteria decision making problems in a more intelligent and more flexible manner.

The rest of this paper is organized as follows. In Section 2, we briefly review some multicriteria decision making methods (Bellman & Zadeh, 1970; Yager, 1988, 1996, 2004a, 2004b) and point out the drawbacks of the methods presented in (Yager, 2004a). In Section 3, we present a new method for handling prioritized multicriteria decision making problems. We also show that the proposed method can overcome the drawbacks of the methods presented in Yager (2004a). In Section 4, we present a generalized prioritized multicriteria decision making method. We also show that it can overcome the drawbacks of the methods presented in Yager (2004a). In Section 5, an example is used to illustrate how the proposed generalized prioritized multicriteria decision making method can handle multicriteria decision making problems with respect to different decision maker's requests. We also make a comparison of the experimental results of the proposed generalized prioritized multicriteria decision making method with Yager's methods (2004a). The conclusions are discussed in Section 6.

2. Preliminary

A multicriteria decision making problem consists of a set A of alternatives, $A = \{A_1, A_2, \ldots, A_m\}$, and a set C of criteria, $C = \{C_1, C_2, \ldots, C_n\}$, to evaluate each alternative and select the best one among them. Bellman and Zadeh (1970) suggested that each criterion C_i can be represented as a fuzzy subset C_i over the set of alternatives A, where $C_i(A_j)$ denotes the degree of satisfaction of an alternative A_j with respect to the criterion C_i and $C_i(A_j) \in [0,1]$. In (Bellman & Zadeh, 1970), an aggregation function F is used to aggregate each $C_i(A_j)$ into an overall degree of satisfaction $D(A_j)$ of each alternative A_j with respect to the set of criteria C, shown as follows:

$$D(A_j) = F(C_1(A_j), C_2(A_j), \cdots, C_n(A_j)). \tag{1}$$

Then, the best solution is the alternative A^*, where $D(A^*) = \underset{A_i \in A}{\mathrm{Max}}\, D(A_i)$. Bellman and Zadeh (1970) suggested an aggregation function as follows:

$$F(C_1(A_j), C_2(A_j), \cdots, C_n(A_j)) = \underset{C_i \in C}{\mathrm{Min}}\, C_i(A_j). \tag{2}$$

Essentially, it assumes that the decision maker desires all the criteria to be satisfied by a good solution. If the decision maker only desires one of the criteria to be satisfied by a good solution, then the aggregation is as follows:

$$F(C_1(A_j), C_2(A_j), \cdots, C_n(A_j)) = \underset{C_i \in C}{\mathrm{Max}}\, C_i(A_j). \tag{3}$$

2.1. OWA operators

Yager (1988) proposed the ordered weighted averaging (OWA) operators. The OWA operator is defined by

$$F(C_1(A), C_2(A), \cdots, C_n(A)) = \sum_{j=1}^{n} w_j C_{\mathrm{index}(j)}(A), \tag{4}$$

where index(j) is the index of the criteria in which alternative A has the jth largest degree of satisfaction. That is, $C_{\mathrm{index}(j)}(A)$ is the jth largest criterion of $C_1(A), C_2(A), \cdots, C_n(A)$; w_j denotes the weight of criterion $C_{\mathrm{index}(j)}(A)$, $w_j \in [0,1]$, and $\sum_{j=1}^{n} w_j = 1$. Yager (1988) pointed out that the OWA operator is a mean operator and it has the following properties:

(1) *Monotonicity:*

$$F(C_1(A), C_2(A), \cdots, C_n(A)) \geqslant F(\overline{C_1(A)}, \overline{C_2(A)}, \cdots, \overline{C_n(A)}),$$
$$\text{for all } i, C_i(A) \geqslant \overline{C_i(A)}.$$

(2) *Symmetry:* For any permutation π,

$$F(C_{\pi(1)}(A), C_{\pi(2)}(A), \cdots, C_{\pi(n)}(A)) = F(C_1(A), C_2(A), \cdots, C_n(A)).$$

(3) *Bounded:* $\min_j C_i(A) \leqslant F(C_1(A), \cdots, C_n(A)) \leqslant \max_j C_j(A)$.

(4) *Idempotency:* If $C_i(A) = a$ for all i, then

$$F(C_1(A), C_2(A), \cdots, C_n(A)) = F(a, a, \cdots, a) = a.$$

The weights are parameters which result in different types of aggregation. When $w_1 = 1$ and $w_i = 0$ for all $j \neq 1$, the OWA operator performs the "Max" operation; when $w_n = 1$ and $w_i = 0$ for all $j \neq n$, the OWA operator performs the "Min" operation; when $w_i = 1/n$ for all j, the OWA operator performs the "Averaging" operation.

2.2. Quantifier-guided aggregation

Yager (1996) proposed an approach to obtain the weights of criteria using linguistic quantifiers (Zadeh, 1983), where the decision maker can specify "Q criteria need to be satisfied for a good solution", where Q is a linguistic quantifier, such as "Most", "At least half" and "Average". Any linguistic quantifier Q can be expressed as a fuzzy set, where Q (r) denotes the degree of satisfaction of r with respect to the concept conveyed by the term Q. The linguistic quantifiers satisfy the following properties:

(1) $Q(0) = 0$,
(2) $Q(1) = 1$,
(3) If $r_1 > r_2$, then $Q(r_1) \geqslant Q(r_1)$.

Using linguistic quantifier Q, the weight of criteria C_j is obtained as follows:

$$w_j = Q\left(\frac{j}{n}\right) - Q\left(\frac{j-1}{n}\right), \quad j = 1, 2, \cdots, n. \tag{5}$$

Yager (1996) also proposed an approach to deal with quantifier-guided aggregation when each criterion has an importance associated with it. Assume that the importance associated with the criterion C_i is a_i, where $a_i \in [0,1]$, then the weight of criterion C_j is obtained as follows:

$$w_j = Q\left(\frac{S_j}{S_n}\right) - Q\left(\frac{S_{j-1}}{S_n}\right), \quad j = 1, 2, \cdots, n, \tag{6}$$

where $S_j = \sum_{k=1}^{j} a_{\mathrm{index}(k)}$ and $a_{\mathrm{index}(k)}$ denotes the importance of the criterion $C_{\mathrm{index}(k)}$.

2.3. Prioritized multicriteria decision making

In the following, we briefly review the prioritized multicriteria decision making methods presented by Yager (2004a). Yager considered the problem of multicriteria decision making in the situation where a prioritization of criteria exists. He finds that the weights associated with the lower priority criteria are related to the degree of satisfaction with respect to the higher priority criteria and proposed a formulation for the aggregation of prioritized criteria. Assume that there are n criteria $C_1, C_2, \ldots, C_n$, where the