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Common sets of weights as summaries of DEA profiles of weights: With an application to the ranking of professional tennis players

Nuria Ramón, José L. Ruiz *, Inmaculada Sirvent

Centro de Investigación Operativa, Universidad Miguel Hernández, Avd. de la Universidad, s/n, 03202 Elche, Alicante, Spain

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ABSTRACT

In this paper, we propose a DEA approach aimed at deriving a common set of weights (CSW) to be used to the ranking of decision making units (DMUs). The idea of this approach is to minimize the deviations of the CSW from the DEA profiles of weights without zeros of the efficient DMUs. This minimization reduces in particular the differences between the DEA profiles of weights that are chosen, so the CSW proposed is a representative summary of such DEA weights profiles. We use several norms to the measurement of such differences. As a result, the CSWs derived are actually different summaries of the chosen DEA profiles of weights like their average profile of their median profile. This approach is illustrated with an application to the ranking of professional tennis players.

1. Introduction

Data envelopment analysis (DEA), as introduced in Charnes, Cooper, and Rhodes (1978), is a methodology for the assessment of relative efficiency of a set of decision making units (DMUs) that use several inputs to produce several outputs. For each DMU, it provides efficiency scores in the form of a ratio of a weighted sum of the outputs to a weighted sum of the inputs. One of the most appealing aspects of this methodology is that we do not need to a priori know exactly the values of the involved weights; these are specified trying to show the unit under assessment in its best possible light. DEA provides weights that are DMU-specific, and therefore it allows for individual circumstances of operation of the DMUs. Aside from the factors affecting performance considered in the efficiency analysis, there are often considerable variations in goals, policies, etc., among DMUs, which may justify the different weights for the same factor. The variation in weights in DEA may be thus justified by the different circumstances under which the DMUs operate, and which are not captured by the chosen set of inputs and outputs factors (see Roll, Cook, and Golany (1991) for discussions).

There are, however, situations in which the different DMUs experience similar circumstances and, therefore, using input and output weights that differ substantially across DMUs may not be warranted. When that is the case, both the inputs and the outputs should be aggregated by using weights that are common to all the DMUs. Common set of weights (CSW, as first denoted in Roll et al. (1991)) is the usual approach in engineering and in most economic efficiency analyses. It has the appeal of a fair and impartial evaluation in the sense that each variable is attached the same weight in the assessments of all the DMUs. Nevertheless, the choice itself of such weights often raises serious difficulties, and in many cases there is no universally agreed-upon the weights to be used as pointed out in Doyle and Green (1994).

It should also be noted that, unlike DEA, CSW allows us to rank the DMUs. The fact that DEA uses different profiles of weights in the assessments of the different DMUs makes impossible to derive an ordering of the units based on the resulting efficiency scores. Moreover, poor discrimination is often found in the assessment of performance with DEA models, since many of the DMUs are classified as efficient or are rated near the maximum efficiency score. This can also be avoided with a CSW. See Adler, Friedman, and Sinuany-Stern (2002), which provides a survey of ranking methods in the context of DEA. See also Angulo-Meza and Estellita Lins (2002) and Podinovski and Thanassoulis (2007), and also the previously mentioned paper by Adler et al., which review the problem of improving discrimination in DEA.

In this paper we propose a DEA approach to derive a CSW to be used to the ranking of DMUs. The basic idea is to determine such CSW by minimizing its deviations from the DEA profiles of weights that do no have zeros of the efficient DMUs. DEA models do not require prior information and provide weights by only using that contained in the data. The efficient DMUs play an important role as referents in the assessments of the remaining units and their weights represent relative value systems of the inputs and outputs involved that make them be rated as efficient. In particular, their optimal solutions without zeros guarantee that no variable is ignored in the assessments of the efficiency. In minimizing the deviations of the CSW from the DEA profiles of weights that are chosen we are also implicitly reducing the differences between such DEA

* Corresponding author.
E-mail address: jlruiz@umh.es (J.L. Ruiz).

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weights profiles among themselves, so the proposed CSW is a representative summary of them. We implement this approach by using three different weighted norms to the measurement of the differences between the CSW and the DEA profiles of weights: the L_1, L_2 and L_∞ norms. It is shown that when the L_1-norm is used the profile of weights that is most similar to those of the DEA efficient DMUs, i.e., the CSW proposed, is the median profile of the DEA profiles of weights of the efficient DMUs, while with the L_2-norm the CSW is the average of such DEA weights profiles. Roll et al. (1991) have already proposed as CSW to take the average (or some other central measure) of DEA weights for each factor, and also to apply some weighting technique to those weights (see also Roll & Golany, 1993). And more recently, Wang and Chin (2010a) have proposed a CSW as the average of the profiles of weights provided by the so-called "neutral" model used in the cross-efficiency evaluation. However, we note that the choice of DEA weights in these papers is not made following a criterion that makes the average of such weights profiles be a suitable approach.

Since our approach provides a CSW as a summary of DEA weights we focus on the choice of the DEA profiles of weights that are to be summarized. To be specific, we look for the CSW that is most similar to the DEA weights profiles that are chosen. There are, however, other DEA approaches aimed at finding a CSW which deal with the efficiency scores that result from the weights that are proposed. For example, Roll and Golany (1993) suggest as CSWs those resulting of either maximizing the average efficiency of all the DMUs or maximizing the number of efficient DMUs (see also Ganley & Cubbin, 1992 for a similar approach). Other approaches are based on the idea of minimizing the differences between the DEA efficiency scores and those obtained with the CSW (note that the DEA efficiency scores are greater than or equal to those obtained relative to a CSW): Kao and Hung (2005) derive a family of CSW's by minimizing the generalized family of distance measures, Despotis (2002) minimizes a convex combination of these deviations measured in terms of a couple of distances in such family, Cook and Zhu (2007) also deal with these distances but relax the objective to groups of DMUs which operate in similar circumstances and Liu and Peng (2008, 2009) deal with deviations regarding the total input virtual and the total output virtual (see also Jahanshahloo, Hosseinzadeh Lotfi, Khanmohammadi, Kazemimanesh, & Rezaie, 2010 for a related approach).

We illustrate the use of the proposed approach with an application to the assessment and ranking of professional tennis players. The use of the CSW proposed allows us to determine an overall index of performance of the players by aggregating into a single value the "statistics" regarding the different aspects of their game. The values of these indexes are used to derive a ranking of players, which provides an insight into the efficiency performance of their game. We believe that this is useful information that can complement that provided by the ATP (the Association of Tennis Professionals) ranking which is concerned with the competitive performance of the players.

The paper unfolds as follows: in Section 2 we develop in different subsections the models that make the choice of DEA weights profiles among alternate optima and provide the corresponding CSWs for different norms used to the measurement of the differences between them. Section 3 includes an application of the proposed approach to the ranking of professional tennis players. Section 4 concludes.

2. Common sets of weights as the profiles most similar to the DEA profiles of weights

Throughout the paper we assume that we have n DMUs which use m inputs to produce s outputs. The purpose is to find a CSW, $(v_1,\ldots,v_m,u_1,\ldots,u_s)$, to be used in the calculation of an efficiency score for each of the DMUs in the form

$$E_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}, \quad j=1,\ldots,n. \tag{1}$$

To do it, we propose here a DEA approach that is based on the idea of minimizing the deviations of the CSW from DEA profiles of weights provided by the CCR model for the efficient DMUs. We confine the attention to these DMUs, in particular because it can be ensured for them a choice of weights without zeros. In minimizing these deviations we make a choice of weights among the alternate optima in the CCR model for the efficient DMUs which also reduces the differences between the DEA weights profiles that are selected, so the CSW proposed is a representative summary of such profiles of weights.

The DEA efficiency score and the associated weights for a given DMU_0 are, respectively, the value and the optimal solutions of the following problem

$$\begin{aligned} \text{Max} \quad & \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}}, \\ \text{s.t.} \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, \quad j=1,\ldots,n, \\ & v_i, u_r \geqslant 0 \quad \forall i, r. \end{aligned} \tag{2}$$

This problem is called the CCR DEA model. It can converted into the following linear problem, called the dual multiplier formulation, by using the results on linear fractional problems in Charnes and Cooper (1962)

$$\begin{aligned} \text{Max} \quad & \sum_{r=1}^{s} u_r y_0, \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{i0} = 1, \\ & -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} \leqslant 0, \quad j=1,\ldots,n, \\ & v_i, u_r \geqslant 0 \quad \forall i, r. \end{aligned} \tag{3}$$

To minimize the deviations of the CSW, $(v_1,\ldots,v_m,u_1,\ldots,u_s)$, from the profiles of weights of the DMU_d's in E (the set of efficient DMUs) provided by the CCR model we use different measures of similarity. If, for example, the following weighted L_1-norm of the vector of differences between the CSW and a DEA profile of weights $(v_1^d,\ldots,v_m^d,u_1^d,\ldots,u_s^d)$ of a given DMU_d in E

$$\sum_{i=1}^{m} |v_i^d - v_i| \bar{x}_i + \sum_{r=1}^{s} |u_r^d - u_r| \bar{y}_r, \tag{4}$$

is used as measure of their similarity, where $\bar{x}_i,\ i=1,\ldots,m$, and $\bar{y}_r,\ r=1,\ldots,s$, are the averages of input i and output r, respectively, across the efficient DMUs, then the CSW we propose is the optimal solution for the variables $(v_1,\ldots,v_m,u_1,\ldots,u_s)$ of the following problem

$$\begin{aligned} \text{Min} \quad & \tfrac{1}{\alpha}\left\{\sum_{d\in E}\left(\sum_{i=1}^{m} |v_i^d - v_i|\bar{x}_i + \sum_{r=1}^{s} |u_r^d - u_r|\bar{y}_r\right)\right\}, \\ \text{s.t.} \quad & -\sum_{i=1}^{m} v_i^d x_{ij} + \sum_{r=1}^{s} u_r^d y_{rj} \leqslant 0, \quad j, d \in E, && (5.1) \\ & -\sum_{i=1}^{m} v_i^d x_{id} + \sum_{r=1}^{s} u_r^d y_{rd} = 0, \quad d \in E, && (5.2) \\ & v_i^d \bar{x}_i \geqslant 1, \quad i=1,\ldots,m;\ d \in E, && (5.3) \\ & u_r^d \bar{y}_r \geqslant 1, \quad r=1,\ldots,s;\ d \in E, && (5.4) \\ & \sum_{i=1}^{m} v_i^d \bar{x}_i + \sum_{r=1}^{s} u_r^d \bar{y}_r = \alpha, \quad d \in E, && (5.5) \\ & v_i \bar{x}_i \geqslant 1, \quad i=1,\ldots,m, && (5.6) \\ & u_r \bar{y}_r \geqslant 1, \quad r=1,\ldots,s, && (5.7) \\ & v_i^d, u_r^d, v_i, u_r, \alpha \geqslant 0, \quad \forall i, r, d. \end{aligned} \tag{5}$$