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An exact approach for the Blocks Relocation Problem

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ABSTRACT

The Blocks Relocation Problem seeks to find the shortest sequence of movements to retrieve a set of homogeneous blocks placed in a two-dimensional storage according to a predefined order. In this paper we analyze an optimization model recently published in the literature. We illustrate that this model reports infeasible solutions and does not guarantee the optimality of the achieved solutions in some cases. In order to overcome this fact, we propose an alternative optimization model. However, its high computational consumption of temporal and space resources in large environments encourages us to also develop a branch and bound algorithm to solve realistic scenarios to optimality. This algorithm includes an intelligent strategy to explore the most promising nodes in the underlying tree. Additionally, its performance can be easily adapted to report high-quality solutions at the expense of sacrificing the optimality guarantee. In contrast to previous exact proposals, the computational experiments reveal the high efficiency of the branch and bound algorithm, reporting optimal solutions for the most widely extended benchmark suite from the related literature through short computational times.

1. Introduction

Improving the management of supply chains is an increasingly cherished goal for transportation companies. The major reason is that it brings down the transportation costs and enhances the utilization of resources. As discussed by Breiter, Hegmanns, Hellingrath, and Spinler (2009), this field has gained increasing interest in the research community over the last decades. In this environment, the warehouses are large facilities dedicated to store freights temporarily until their subsequent retrieval within a supply chain. Their relevance lies in their essential role as intermediate points in the flow of freights, from producers to customers. In fact, the suitability of the warehouse operations has a high impact not only on the performance of the warehouse, but also on the whole supply chain. See the work by Melo, Nickel, and da Gama (2009) to obtain a comprehensive review of the management of supply chains.

The freights in a warehouse are usually packaged into robust load units (Gu, Goetschalckx, & McGinnis, 2010). Such is the case, for instance, of containers on the yard of a maritime container terminal. From a general standpoint, each load unit is termed *block*. For the sake of simplicity, the most widespread storage strategy used in warehouses is the block stacking. In this strategy, blocks with similar characteristics (*i.e.*, dimensions, weights, etc.) are piled up one on each other in stacks, in such a way that they make up two-dimensional storage structures denominated as bays (Gudehus & Kotzab, 2009). The number of tiers is usually established by the physical features of the warehouse, the load strength (crushability), and the stability of the stacks. In this regard, the location of a block is then given by the bay, stack, and tier in which is currently placed (Park & Kim, 2010).

The retrieval of blocks from their storage locations in a warehouse is known as an order-picking (de Koster, Le-Duc, & Roodbergen, 2007). As discussed by Tompkins, White, Bozer, and Tanchoco (2010), order-pickings constitute the most labor-intensive and time-consuming processes in warehouse logistics. Fulfilling the order-pickings efficiently is considered by several authors as the main indicator of the competitiveness of a warehouse (Chan & Chan, 2011). In this regard, the current position of the blocks is the most influencing factor on the time required to fulfil a certain order-picking. That is, the Last In First Out (LIFO) policy stemming from the block stacking gives rise to that those blocks currently placed at the top of the stacks can be accessed directly, whereas those buried below other ones must be firstly freed up and then retrieved. Relocation movements must be performed in order to access a block placed below other ones, in such a way that those currently placed above it are moved toward alternative stacks in the warehouse (Caserta, Schwarze, & Voß, 2011).

The block relocation movements are unproductive, and therefore should be minimized due to the fact that they increase the

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labor costs and impair the customer satisfaction (Dekker, Voogd, & Asperen, 2007). Minimizing the number of relocation movements in a two-dimensional storage structure is known as Blocks Relocation Problem (BRP). With the goal of addressing the BRP, efficient planning strategies must be developed (Choe, Park, Oh, Kang, & Ryu, 2011).

The main motivation to address the BRP from a research standpoint arises from the fact that reducing the number of relocation movements in a warehouse could improve its performance drastically, and then increase the economical savings. As indicated by Pazour and Carlo (2015), there are documented savings of \$500.000 per year after efficient planning strategies have been set up, especially in large facilities containing lots of stock keeping units. In the remaining cases, average savings of an 8% to 15% in picking and replenishment labor are common.

The relevance and economical impact of the operations carried out in supply chains encourage the development of intelligent systems in which efficient planning strategies are used as knowledge (Metaxiotis, Askounis, & Psarras, 2002) to support the decision making process. Thus, the planning strategies are aimed at reporting feasible sequences of movements to perform while the constraints of the BRP are satisfied. In these systems, the decision maker controls the inputs of the available strategies whereas the optimization techniques are considered as black boxes (Ganapathy, Prabhala, Narayanan, Hill, & Gallimore, 2011; Schneider, Narayanan, & Patel, 2000; Wang & Shen, 1989).

According to the previous discussion, the main contributions of this paper are described as follows:

1. Presenting an optimization model for the BRP that overcomes several shortcomings of the model termed BRP-II (Caserta, Schwarze, & Voß, 2012). As discussed in Section 4 and illustrated in Section 6.1, in contrast to our model, BRP-II reports infeasible solutions and does not guarantee the optimality of the achieved solutions in some cases. The reasons are found in that (i) several blocks currently placed above the next to retrieve cannot be relocated to the same destination stack and (ii) the blocks placed below the next to retrieve can be relocated.
2. Developing a branch and bound algorithm aimed at solving the BRP to optimality by means of short computational times. This algorithm includes an intelligent strategy to explore the most promising nodes in the underlying tree. Thus, the performance of the branch and bound algorithm can be easily adapted to report high-quality solutions at the expense of sacrificing the optimality guarantee.

The remainder of this paper is organized as follows. We firstly introduce the BRP in Section 2. Then, in Section 3 we review the related works from the literature. Afterwards, we proceed with an optimization model for the BRP in Section 4. In Section 5, we propose a branch and bound algorithm for solving the BRP to optimality. The computational experiments carried out are discussed in Section 6. Finally, in Section 7 we finish the paper with the main concluding remarks and several directions for further research.

2. Blocks Relocation Problem

The Blocks Relocation Problem (BRP) is a combinatorial optimization problem belonging to the $\mathcal{NP}$-hard class (Caserta et al., 2012) that can be described as follows. Given a set of N homogeneous blocks placed in a two-dimensional storage composed of S stacks and T tiers, the goal of the BRP is to find the shortest sequence of movements aimed at retrieving all the blocks one after the other according to their decreasing priority order. The capacity of the storage is derived from the number of stacks and tiers, that is, $C = S \times T$. Each block c has a given priority denoted as $p(c)$. Without loss of generality, it is assumed that the retrieval order is defined by following the block priorities, in such a way that, the block with the highest priority, 1, must be retrieved before block 2; block 2 must be retrieved before block 3; and so forth, until all the blocks are retrieved. In this context, c^* denotes the next block to retrieve.

In the following some notation is introduced to ease the perusal of the remainder of this paper. Given a block c, let $s(c)$ and $t(c)$ be the stack and the tier in which c is currently placed, respectively. The number of blocks placed in the stack s is denoted as $h(s)$. Additionally, the set of blocks currently placed above c is defined as follows:

$$O(c) = \{c' \,|\, (s(c') = s(c)) \wedge (t(c') > t(c))\}. \tag{1}$$

Finally, the highest priority of a block in the stack s is denoted by $max(s)$ and defined as follows:

$$max(s) = \begin{cases} \min\{p(c) \,|\, s(c) = s\}, & \text{if } t(s) > 0, \\ N + 1, & \text{otherwise.} \end{cases} \tag{2}$$

As indicated by Expósito-Izquierdo, Melián-Batista, and Moreno-Vega (2014), the blocks can be classified according to their priorities and the slots where they are placed. In this regard, a non-located block is that placed at a higher tier than another one in the same stack with a higher priority. See striped blocks in the example depicted in Fig. 1. Let $\Omega(s)$ be the set of non-located blocks in stack s, defined as follows:

$$\Omega(s) = \{c \,|\, (s(c) = s) \wedge \exists\, c' : (s(c') = s) \wedge (t(c') < t(c)) \wedge (p(c') < p(c))\}, \quad \forall 1 \leqslant s \leqslant S. \tag{3}$$

The set of non-located blocks into the two-dimensional storage is defined as

$$\Omega = \bigcup_{1 \leqslant s \leqslant S} \Omega(s). \tag{4}$$

On the other hand, a well-located block is that not placed above any other block with a higher priority in the same stack. The set of well-located blocks is formally defined as follows:

$$\Upsilon(s) = \{c \,|\, (s(c) = s) \wedge (c \notin \Omega(s))\}, \quad \forall 1 \leqslant s \leqslant S. \tag{5}$$

Examples of well-located blocks are those with white background in Fig. 1.

The retrieval of blocks can be performed by following two general types of movements:

- *Retrieval*. The block c^* currently placed at the top of some stack is moved outside the storage. Each retrieval movement is represented as a pair $(a, -)$, where a is the source stack.
- *Relocation*. A block placed at the top of a stack is moved toward the top of another one (Kim & Hong, 2006). The target stack must contain at least one empty slot. Each relocation movement is represented as a pair (a, b), where a is the source stack whereas b is the target stack.

Up to now, the literature contains the following main variants of the BRP:

- *Unrestricted BRP*. The source stack of each relocation movement can be each one of the stacks.
- *Restricted BRP*. The source stack of each relocation movement is limited to $s(c^*)$. It is worth mentioning that this paper is restricted to this variant.